TABLES

AND

TRACTS,

RELATIVE TO

Several ARTS and SCIENCES.

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ADVERTISEMENT.

THE uſefulneſs of numerical Tables, in the practical arts and ſciences, is ſo univerſally acknowledged, that the preſent publication ſcarce needs an apology. By their means we ſave a vaſt deal of time and labour: witneſs the facility with which the operations of Trigonometry, and the more difficult queſtions of arithmetic, are now performed, when compared with the operoſe and diſcouraging methods of computation uſed in ancient times.

No wonder therefore that the Trigonometrical Tables, and thoſe of Logarithms, reduced to the moſt perfect form, by the ſucceſſive labours of learned and ingenious men, are in every body's hands. But ſtill there are many particular Tables and Tracts, relative to uſeful Arts and Sciences, which lie ſcattered in different volumes, ſome in print and ſome in mauuſcript, to which many curious perſons cannot always have

 ready

ready acceſs. Such of theſe as the author judged would be moſt acceptable to the public, he hath collected into this manual, together with a few eaſy rules and examples directing their uſe. To theſe he hath added ſeveral of his own: and, throughout the Tables, he hath taken all poſſible care that the numbers ſhould be correct.

But although, on reviſing the book after it was printed, he has not found any errors in the Tables; yet he has found ſome in the other parts of it: theſe, he has put down after the Contents; and begs that the candid Reader will excuſe, and correct them.

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ERRATA.

ERRATA.

Pag. 112, *l.* 5 from the bottom, *for* rround *read* round. *Pag.* 121, *l.* 11, *for* equation time *read* equation of time. *Pag.* 142, *l.* 1, *for* multiples *read* multipliers. *Pag.* 144, in *l.* 7 and 8, leave out the parenthesis, and all that is within it. *Pag.* 144, *l.* 11, *for* multiples *read* multipliers. *Pag.* 167, *l.* 3 from the bottom, *for* worrthy *read* worthy. *Pag.* 179, *l.* 3 *and* 4 from the bottom, for 4640 and 4618, *read* 6440 *and* 6418.

The Reader is desired to observe, that from the 16*th line of* page 139 *to the end of* page 140, *the whole ought to be as follows.*

3960 hours (the conjunction of A, B, C, and D) multiplied by 2, is equal to 144 hours (or $\frac{de}{d-e}$) multiplied by 55; equal to 7920 hours, for the time of the next conjunction of A, B, C D, and E, after their first setting out together. And

7920 hours (the last mentioned conjunction) multiplied by 7, is equal to 112 hours (the conjunction of E and F, or $\frac{ef}{e-f}$) multiplied by 495; equal to 55440 hours, the time when all the six hands A, B, C, D, E and F will be in conjunction again, after the instant of their first setting out together, from a conjunction at any given point of the Dial-plate, and all moving round the same way, in the times abovementioned.

Now, as it will require 55440 hours (or 2310 days) to bring all these hands together again, after their first setting out together; divide 55440 hours by the number of hours in which each hand goes round, and the quotients will shew that A has made 2310 revolutions, B 2520, C 2772, D 3080, E 3465, and F 3960. And, at the end of so many more revolutions of each hand, they will all be in conjunction again; and so on continually.

TABLE I.

The mean time of New Moon in January, according to the Old Stile.

Julian Years.	New Moon.			Sun's Anomaly.			Moon's Anomaly.			Sun from Node.		
	D.	H.	M.	S.	°	′	S.	°	′	S.	°	′
1700	8	14	45	6	21	14	0	0	55	4	13	9
01	27	12	17	7	10	7	11	6	32	5	21	52
02	16	21	6	6	29	22	9	16	20	5	29	55
03	6	5	55	6	18	38	7	26	8	6	7	57
04	24	3	27	7	7	1	7	1	45	7	16	41
05	13	12	16	6	26	16	5	11	33	8	24	43
06	2	21	4	6	15	32	3	21	21	8	2	46
07	21	18	37	7	3	35	2	26	58	9	11	29
08	10	3	25	6	23	11	1	6	46	9	19	32
09	29	0	58	7	11	33	0	12	23	10	28	15
1710	18	19	47	7	0	48	10	22	12	11	6	18
11	7	18	35	6	20	3	9	2	0	11	14	20
12	25	16	8	7	8	26	8	7	37	0	23	3
13	15	0	56	6	27	42	6	17	25	1	1	6
14	4	9	45	6	16	57	4	27	13	1	9	9
15	23	7	17	7	5	20	4	2	50	2	17	52
16	11	16	6	6	24	35	2	12	38	2	25	54
17	1	0	54	6	13	51	0	22	26	3	3	37
18	19	22	27	7	2	13	11	28	3	4	12	40
19	9	7	16	6	21	29	10	7	51	4	20	43
1720	27	4	48	7	9	51	9	13	28	5	29	26
21	16	13	37	6	29	7	7	23	16	6	7	29
22	5	22	25	6	18	23	6	3	4	6	15	31
23	24	19	58	7	6	45	5	8	41	7	24	14
24	13	4	46	6	26	1	3	18	29	8	2	17
25	2	13	35	6	15	17	1	28	17	8	10	20
26	21	11	8	7	3	39	1	3	54	9	19	3
27	10	19	56	6	22	55	11	13	42	9	27	6
28	28	17	29	7	11	17	10	19	19	11	5	49
29	18	2	17	7	0	33	8	29	7	11	13	51
1730	7	11	6	6	19	49	7	8	55	11	21	54
31	26	8	38	7	8	11	6	14	33	1	0	37
32	14	17	27	6	27	27	4	24	20	1	8	40

TABLE I. *continued.*

Old Stile.

Julian Years.	New Moon D.	H.	M.	Sun's Anomaly. S.	°	′	Moon's Anomaly. S.	°	′	Sun from Node. S.	°	′
1733	4	2	16	6	16	43	3	4	9	1	16	43
34	22	23	48	7	5	5	2	9	46	2	25	26
35	12	8	37	6	14	21	0	19	34	3	3	28
36	0	17	25	6	13	36	10	29	22	3	11	31
37	19	14	58	7	1	58	10	4	59	4	20	14
38	8	23	47	6	21	14	8	14	47	4	28	17
39	27	21	19	7	9	36	7	20	24	6	7	0
1740	16	6	8	6	28	52	6	0	12	6	15	3
41	5	14	56	6	18	9	4	10	0	6	23	5
42	24	12	29	7	6	30	3	15	37	8	1	48
43	13	21	18	6	25	46	1	25	25	4	9	51
44	2	6	6	6	15	2	0	15	13	8	17	54
45	21	3	39	7	3	24	11	10	50	9	26	37
46	10	12	27	6	22	39	9	20	38	10	4	40
47	29	10	10	7	11	2	8	26	16	11	13	23
48	17	18	48	7	0	17	7	6	4	11	21	25
49	7	3	37	6	19	33	5	15	52	11	29	28
1750	26	1	10	7	7	55	4	21	29	1	8	11
51	15	9	58	6	27	11	3	1	17	1	16	14
52	3	18	47	6	16	27	1	11	5	1	24	17
53	22	16	19	7	4	49	0	16	42	3	3	0
54	12	1	8	6	24	4	10	26	30	3	11	2
55	1	9	56	6	13	21	9	6	18	3	19	5
56	19	7	29	7	1	43	8	11	55	4	27	48
57	8	16	17	6	20	59	6	21	43	5	5	51
58	27	13	50	7	9	21	5	27	20	6	14	34
59	16	22	39	6	28	36	4	7	8	6	22	37
1760	5	7	27	6	17	52	2	16	56	7	0	39
61	24	5	0	7	6	14	1	22	33	8	9	22
62	13	13	48	6	25	30	0	2	21	8	17	25
63	2	22	36	6	14	46	10	12	9	8	25	28
64	20	20	9	7	3	8	9	17	46	10	4	11
65	10	4	58	6	22	24	7	27	35	10	12	14
66	29	2	30	7	10	46	7	3	12	10	20	57

TABLE

TABLE I. *concluded.*

Old Stile.

Julian Years.	New Moon.			Sun's Anomaly.			Moon's Anomaly.			Sun from Node.		
	H.	M.	S.	S.	°	′	S.	°	′	S.	°	′
1767	18	11	19	7	0	2	5	13	0	11	28	59
68	6	20	8	6	19	18	3	22	48	0	7	2
69	25	17	40	7	7	40	2	28	25	1	15	45
1770	15	2	29	6	26	56	1	8	13	1	23	48
71	4	11	17	6	16	11	11	18	1	2	1	51
72	22	8	50	7	4	33	10	23	38	3	10	34
73	11	17	38	6	23	49	9	3	26	3	18	36
74	1	2	27	6	13	5	7	13	14	3	26	39
75	20	0	0	7	1	27	6	18	51	5	5	22
76	8	8	48	6	20	43	4	28	39	5	13	25
77	27	6	21	7	9	5	4	4	16	6	22	8
78	16	15	9	6	28	21	2	14	4	7	0	11
79	5	23	58	6	17	37	0	23	52	7	8	13
1780	23	21	30	7	5	59	11	29	29	8	16	56
81	13	6	19	6	25	15	10	9	17	8	24	59
82	2	15	7	6	14	30	8	19	5	9	3	2
83	21	12	40	7	2	52	7	24	42	10	11	45
84	9	21	29	6	22	8	6	4	31	10	19	48
85	28	19	1	7	10	30	5	10	8	11	28	31
86	18	3	50	6	29	46	3	19	56	0	6	33
87	7	12	38	6	19	2	1	29	44	0	14	36
88	15	10	11	7	7	24	1	5	21	1	23	19
89	14	18	59	6	26	40	11	15	9	2	1	22
1790	4	3	48	6	15	55	9	24	57	2	9	25
91	23	1	21	7	4	18	9	0	34	3	18	8
92	11	10	9	6	23	33	7	10	22	3	26	10
93	0	18	58	6	12	49	5	20	10	4	4	13
94	19	16	30	7	1	11	4	25	47	5	12	56
95	9	1	19	6	20	27	3	5	35	5	20	59
96	26	22	51	7	8	49	2	11	12	6	29	42
97	16	7	40	6	28	5	0	21	0	7	7	45
98	5	16	29	6	17	21	11	0	48	7	15	47
99	24	14	1	7	5	43	10	6	25	8	24	30
1800	12	22	50	6	24	59	8	16	14	9	2	33

Table II.

Mean New Moon in January, New Stile.

Greg. Years.	New Moon			Sun's Anomaly.			Moon's Anomaly.			Sun from Node		
	D.	H.	M.	S.	°	′	S.	°	′	S.	°	′
1752	14	18	47	6	16	27	1	11	5	1	24	17
53	4	3	35	6	5	43	11	20	53	2	2	19
54	23	1	8	6	24	5	10	26	30	3	11	2
55	12	9	56	6	13	21	9	6	18	3	19	5
56	30	7	29	7	1	43	8	11	55	4	27	48
57	19	16	17	6	20	59	6	21	43	5	5	51
58	9	1	6	6	10	14	5	1	31	5	13	54
59	27	22	39	6	28	36	4	7	8	6	22	37
1760	16	7	27	6	17	52	2	16	56	7	0	39
61	5	16	16	6	7	8	0	26	44	7	8	42
62	24	13	48	6	25	30	0	2	21	8	17	25
63	13	22	37	6	14	46	10	12	9	8	25	28
64	2	7	25	6	4	2	8	21	57	9	3	31
65	21	4	58	6	22	24	7	27	35	10	12	14
66	10	13	46	6	11	40	6	7	23	10	20	16
67	29	11	19	7	0	2	5	13	0	11	28	59
68	17	20	8	6	19	18	3	22	48	0	17	2
69	7	4	56	6	8	33	2	2	36	0	25	5
1770	26	2	49	6	26	56	1	8	13	1	23	48
71	15	11	17	6	16	11	11	18	1	2	1	51
72	3	20	6	6	5	27	9	27	49	2	9	53
73	22	17	38	6	23	49	9	3	26	3	18	36
74	12	2	27	6	13	5	7	13	14	3	26	39
75	1	11	15	6	2	21	5	23	2	4	4	42
76	19	8	48	6	20	43	4	28	39	5	13	25
77	8	17	37	6	9	59	3	8	27	5	21	28
78	27	15	9	6	28	21	2	14	4	7	0	11
79	16	23	58	6	17	37	0	23	52	7	8	13
1780	5	8	46	6	6	52	11	3	40	7	16	16
81	24	6	19	6	25	15	10	9	17	8	24	59
82	13	15	7	6	14	30	8	19	5	9	3	2
83	2	23	56	6	3	46	6	28	53	9	11	5
84	20	21	29	6	22	8	6	4	31	10	19	48
85	10	6	17	6	11	24	4	14	19	10	27	50
86	29	3	50	6	29	46	3	19	56	0	6	33

Table II. *concluded. New Stile.*

Greg. Years.	New Moon D.	H.	M.	Sun's Anomaly. S.	°	′	Moon's Anomaly. S.	°	′	Sun from Node. S.	°	′
1787	18	12	38	6	19	2	1	29	44	0	14	36
88	6	21	26	6	8	18	0	9	32	0	22	39
89	25	18	59	6	26	40	11	15	9	2	1	22
1790	15	3	48	6	15	56	9	24	57	2	9	25
91	4	12	37	6	5	11	8	4	45	2	17	27
92	22	10	9	6	23	33	7	10	22	3	26	10
93	11	18	58	6	12	49	5	20	10	4	4	13
94	1	3	46	6	2	5	3	29	58	4	12	16
95	20	1	19	6	20	27	3	5	35	5	20	59
96	8	10	7	6	9	43	1	15	23	5	29	2
97	27	7	40	6	28	5	0	21	0	7	7	45
98	16	16	29	6	17	21	11	0	48	7	15	47
99	6	1	17	6	6	37	9	10	37	7	23	50
1800	24	22	50	6	24	59	8	16	13	9	2	33

Table III.

Containing 13½ *mean Lunations.*

Lun.	New Moon D.	H.	M.	Sun's Anomaly. S.	°	′	Moon's Anomaly. S.	°	′	Sun from Node. S.	°	′
1	29	12	44	0	29	6	0	25	49	1	0	40
2	59	1	28	1	28	13	1	21	38	2	1	20
3	88	14	12	2	27	19	2	17	27	3	2	1
4	118	2	56	3	26	25	3	13	16	4	2	41
5	147	15	40	4	25	32	4	9	5	5	3	21
6	177	4	24	5	24	38	5	4	54	6	4	1
7	206	17	8	6	23	44	6	0	43	7	4	42
8	236	5	52	7	22	51	6	26	32	8	5	22
9	265	18	36	8	21	57	7	22	21	9	6	2
10	295	7	20	9	21	3	8	18	10	10	6	42
11	324	20	5	10	20	10	9	13	59	11	7	23
12	354	8	49	11	19	16	10	9	48	0	8	3
13	383	21	33	0	18	22	11	5	37	1	8	43
½	17	18	22	0	14	33	6	12	54	0	15	20

TABLE IV.

Supplemental to Table I. for finding the mean time of New Moon in January, for 6000 years before or after any given year of the 18th Century, according to the Old Stile.

Julian Cent^s.	New Moon.			Sun's Anomaly.			Moon's Anomaly.			Sun from Node.		
	D.	H.	M.	S.	°	′	S.	°	′	S.	°	′
100	4	8	5	0	3	15	8	15	19	4	19	24
200	8	16	10	0	6	29	5	0	37	9	8	48
300	13	0	14	0	9	44	1	15	56	1	28	12
400	17	8	19	0	12	59	10	1	15	6	17	36
500	21	16	24	0	16	13	6	16	33	11	7	0
600	26	0	29	0	19	28	3	1	52	3	26	24
700	0	19	50	11	23	36	10	21	22	7	15	8
800	5	3	55	11	26	51	7	6	40	0	4	31
900	9	11	59	0	0	16	3	21	59	4	23	55
1000	13	20	4	0	3	20	0	7	18	9	13	19
1100	18	4	9	0	6	35	8	22	37	2	2	43
1200	22	12	14	0	9	49	5	7	55	6	22	7
1300	26	20	19	0	13	4	1	23	14	11	11	31
1400	1	15	40	11	17	2	9	12	44	3	0	15
1500	5	53	44	11	20	27	5	28	2	7	19	39
1600	10	7	49	11	23	42	2	13	21	0	9	3
1700	14	15	54	11	26	56	10	28	40	4	28	27
1800	18	23	59	0	0	11	7	13	58	9	17	51
1900	23	8	4	0	3	26	3	29	17	2	7	15
2000	27	16	9	0	6	40	0	14	36	6	26	39
2100	2	11	29	11	10	49	8	4	5	10	15	23
2200	6	19	34	11	14	3	4	19	24	3	4	47
2300	11	3	39	11	17	18	1	4	43	7	24	11
2400	15	11	44	11	20	33	9	20	1	0	13	34
2500	19	19	49	11	23	47	6	5	20	5	2	58
2600	24	3	52	11	27	2	2	20	39	9	22	22
2700	28	11	58	0	0	17	11	5	57	2	11	46
2800	3	7	19	11	4	25	6	25	27	6	0	30
2900	7	15	24	11	7	40	3	10	46	10	19	54
3000	11	23	29	11	10	54	11	26	4	3	9	18

TABLE

TABLE IV. *concluded.*

Julian Cents.	New Moon			Sun's Anomaly.			Moon's Anomaly.			Sun from Node.		
	D.	H.	M.	S.	°	′	S	°	′	S.	°	′
3100	16	7	34	11	14	9	8	11	23	7	28	42
3200	20	15	58	11	17	24	4	26	42	0	18	16
3300	24	23	43	11	20	38	1	12	1	5	7	30
3400	29	7	48	11	23	53	9	27	19	9	26	54
3500	4	3	9	10	28	1	5	16	49	1	15	38
3600	8	11	14	11	1	16	2	2	8	6	5	2
3700	12	19	19	11	4	30	10	17	26	10	24	26
3800	17	3	23	11	7	45	7	2	45	3	13	50
3900	21	11	28	11	11	0	3	18	4	8	3	14
4000	25	19	33	11	14	14	0	3	22	0	22	37
4100	0	14	54	10	18	23	7	22	52	4	11	21
4200	4	22	59	10	21	37	4	8	11	9	0	45
4300	9	7	4	10	24	52	0	23	29	11	20	9
4400	13	15	8	10	28	7	6	8	48	6	9	33
4500	17	23	13	11	1	21	5	24	7	10	28	57
4600	22	7	18	11	4	36	2	9	25	3	18	21
4700	26	15	23	11	7	51	10	24	44	8	7	45
4800	1	10	44	10	11	59	6	14	14	11	26	29
4900	5	18	48	10	15	14	2	29	32	4	15	53
5000	10	2	59	10	18	28	11	14	51	9	5	17
5100	14	10	58	10	21	43	8	0	10	1	24	41
5200	18	19	3	10	24	58	4	15	29	6	14	5
5300	23	3	8	10	28	12	1	0	47	11	3	29
5400	27	11	13	11	1	27	9	16	6	3	22	53
5500	2	6	33	10	5	35	3	15	36	7	11	36
5600	6	14	38	10	8	50	1	20	54	0	1	0
5700	10	22	43	10	12	5	10	16	13	4	20	24
5800	15	6	48	10	15	19	6	21	32	9	9	48
5900	19	14	53	10	18	34	3	6	50	1	29	12
6000	23	22	58	10	21	48	11	22	9	6	18	36

The centurial differences in this Table are equal, but in Lunations themſelves they are not.—The following Table ſhews the centurial variations.

TABLE V.

Variations in the mean times of New and Full Moons for 30 centuries, both before and after the 18th century.

Julian Centuries.	New Moon.			Moon's Anomaly.			Sun from Node.		
	Subtract.			*Add.*			*Subtract.*		
	H.	M.	S.	°	′	″	°	′	″
100	0	0	25	0	0	24	0	0	8
200	0	1	40	0	1	36	0	0	32
300	0	3	45	0	3	36	0	1	12
400	0	6	40	0	6	24	0	2	8
500	0	10	25	0	10	0	0	3	20
600	0	15	0	0	14	24	0	4	48
700	0	20	25	0	19	36	0	6	32
800	0	26	40	0	25	36	0	8	32
900	0	33	45	0	32	24	0	10	48
1000	0	41	40	0	40	0	0	13	20
1100	0	50	25	0	48	24	0	16	8
1200	1	0	0	0	57	36	0	19	12
1300	1	10	25	1	7	36	0	22	32
1400	1	21	40	1	18	24	0	26	8
1500	1	33	45	1	30	0	0	30	0
1600	1	46	40	1	42	24	0	34	8
1700	2	0	25	1	55	36	0	38	32
1800	2	15	0	2	9	36	0	43	12
1900	2	30	25	2	24	24	0	48	8
2000	2	46	40	2	40	0	0	53	20
2100	3	3	45	2	56	24	0	58	48
2200	3	21	40	3	13	36	1	4	32
2300	3	40	25	3	31	36	1	10	32
2400	4	0	0	3	50	24	1	16	48
2500	4	20	25	4	10	0	1	23	12
2600	4	41	40	4	30	24	1	30	8
2700	5	3	45	4	51	36	1	37	12
2800	5	26	40	5	13	36	2	44	32
2900	5	50	25	5	36	24	1	52	8
3000	6	15	0	6	0	0	2	0	0

The numbers in this Table are always to be subtracted from the mean time of New Moon, and Sun's mean distance from the Node, as given by the preceding Tables, and added to the mean Anomaly of the Moon, in all Centuries both before and after the 18th.

TABLE

Table VI.

The days in a common year, reckoned from the beginning of January, and serving (with the foregoing Tables) to find the days of New and Full Moons in all the other months.

Days.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1	1	32	60	91	121	152	182	213	244	274	305	335
2	2	33	61	92	122	153	183	214	245	275	306	336
3	3	34	62	93	123	154	184	215	246	276	307	337
4	4	35	63	94	124	155	185	216	247	277	308	338
5	5	36	64	95	125	156	186	217	248	278	309	339
6	6	37	65	96	126	157	187	218	249	279	310	340
7	7	38	66	97	127	158	188	219	250	280	311	341
8	8	39	67	98	128	159	189	220	251	281	312	342
9	9	40	68	99	129	160	190	221	252	282	313	343
10	10	41	69	100	130	161	191	222	253	283	314	344
11	11	42	70	101	131	162	192	223	254	284	315	345
12	12	43	71	102	132	163	193	224	255	285	316	346
13	13	44	72	103	133	164	194	225	256	286	317	347
14	14	45	73	104	134	165	195	226	257	287	318	348
15	15	46	74	105	135	166	196	227	258	288	319	349
16	16	47	75	106	136	167	197	228	259	289	320	350
17	17	48	76	107	137	168	198	229	260	290	321	351
18	18	49	77	108	138	169	199	230	261	291	322	352
19	19	50	78	109	139	170	200	231	262	292	323	353
20	20	51	79	110	140	171	201	232	263	293	324	354
21	21	52	80	111	141	172	202	233	264	294	325	355
22	22	53	81	112	142	173	203	234	265	295	326	356
23	23	54	82	113	143	174	204	235	266	296	327	357
24	24	55	83	114	144	175	205	236	267	297	328	358
25	25	56	84	115	145	176	206	237	268	298	329	359
26	26	57	85	116	146	177	207	238	269	299	330	360
27	27	58	86	117	147	178	208	239	270	300	331	361
28	28	59	87	118	148	179	209	240	271	301	332	362
29	29		88	119	149	180	210	241	272	302	333	363
30	30		89	120	150	181	211	242	273	303	334	364
31	31		90		151		212	243		304		365

TAB. VII. *First Equation from mean to true Syzygy.*

Argument. Sun's mean Anomaly.

Subtract.

Degrees.	0 Signs. H. M.	1 Sign. H. M.	2 Signs. H. M.	3 Signs. H. M.	4 Signs. H. M.	5 Signs. H. M.	Degrees.
0	0 0	2 3	3 35	4 11	3 40	2 8	30
1	0 4	2 7	3 37	4 11	3 37	2 4	29
2	0 9	2 11	3 39	4 11	3 35	2 0	28
3	0 13	2 14	3 41	4 11	3 33	1 56	27
4	0 17	2 18	3 43	4 11	3 30	1 52	26
5	0 21	2 21	3 45	4 10	3 28	1 48	25
6	0 26	2 25	3 47	4 10	3 26	1 41	24
7	0 29	2 28	3 49	4 10	3 23	1 40	23
8	0 34	2 32	3 51	4 9	3 20	1 36	22
9	0 38	2 35	3 52	4 9	3 18	1 32	21
10	0 43	2 39	3 54	4 8	3 15	1 28	20
11	0 47	2 42	3 56	4 7	3 12	1 23	19
12	0 51	2 45	3 57	4 6	3 9	1 19	18
13	0 55	2 48	3 58	4 6	3 6	1 15	17
14	0 59	2 52	4 0	4 5	3 3	1 11	16
15	1 4	2 55	4 1	4 4	3 0	1 6	15
16	1 8	2 58	4 2	4 3	2 57	1 2	14
17	1 12	3 1	4 3	4 1	2 54	0 58	13
18	1 16	3 4	4 4	4 0	2 51	0 53	12
19	1 20	3 7	4 5	3 58	2 47	0 49	11
20	1 24	3 10	4 6	3 57	2 44	0 44	10
21	1 28	3 12	4 7	3 56	2 41	0 40	9
22	1 32	3 15	4 8	3 54	3 37	0 36	8
23	1 36	3 18	4 8	3 53	2 34	0 31	7
24	1 40	3 20	4 9	3 51	2 30	0 27	6
25	1 44	3 23	4 9	3 49	2 26	0 22	5
26	1 48	3 26	4 10	3 48	2 23	0 18	4
27	1 52	3 28	4 10	3 46	2 19	0 13	3
28	1 56	3 30	4 11	3 44	2 15	0 8	2
29	1 59	3 33	4 11	3 42	2 12	0 4	1
30	2 3	3 35	4 11	3 40	2 8	0 0	0
Deg.	11 Signs.	10 Signs.	9 Signs.	8 Signs.	7 Signs.	6 Signs.	Deg.

Add.

TABLE

TABLE VIII. *Equation of the Moon's mean Anomaly.*

Argument. Sun's mean Anomaly.

Subtract.

Degrees.	0 Signs. ° ′	1 Sign. ° ′	2 Signs. ° ′	3 Signs. ° ′	4 Signs. ° ′	5 Signs. ° ′	Degrees.
0	0 0	0 47	1 22	1 35	1 23	0 48	30
1	0 2	0 48	1 22	1 35	1 22	0 47	29
2	0 3	0 50	1 23	1 35	1 21	0 45	28
3	0 5	0 51	1 24	1 35	1 21	0 44	27
4	0 6	0 52	1 25	1 35	1 20	0 42	26
5	0 8	0 54	1 26	1 35	1 19	0 41	25
6	0 10	0 55	1 27	1 34	1 18	0 39	24
7	0 11	0 56	1 27	1 34	1 17	0 38	23
8	0 13	0 58	1 28	1 34	1 16	0 36	22
9	0 15	0 59	1 28	1 34	1 15	0 35	21
10	0 16	1 0	1 29	1 33	1 14	0 33	20
11	0 18	1 2	1 29	1 33	1 13	0 32	19
12	0 19	1 3	1 30	1 33	1 12	0 30	18
13	0 21	1 4	1 30	1 32	1 10	0 28	17
14	0 23	1 5	1 31	1 32	1 9	0 27	16
15	0 24	1 6	1 31	1 32	1 8	0 25	15
16	0 26	1 7	1 32	1 32	1 7	0 23	14
17	0 27	1 9	1 32	1 31	1 6	0 22	13
18	0 29	1 10	1 33	1 31	1 5	0 20	12
19	0 30	1 11	1 33	1 30	1 3	0 18	11
20	0 32	1 12	1 34	1 30	1 2	0 17	10
21	0 33	1 13	1 34	1 29	1 1	0 15	9
22	0 35	1 14	1 34	1 29	0 59	0 13	8
23	0 37	1 15	1 34	1 28	0 58	0 12	7
24	0 38	1 16	1 34	1 28	0 57	0 10	6
25	0 39	1 17	1 34	1 27	0 55	0 8	5
26	0 41	1 18	1 35	1 26	0 54	0 7	4
27	0 42	1 19	1 35	1 26	0 53	0 5	3
28	0 44	1 20	1 35	1 25	0 51	0 3	2
29	0 45	1 21	1 35	1 24	0 50	0 2	1
30	0 47	1 22	1 35	1 23	0 48	0 0	0
Deg.	11 Signs.	10 Signs.	9 Signs.	8 Signs.	7 Signs.	6 Signs.	Deg.

Add.

TABLE IX. *Second Equation from mean to true Syzygy.*

Argument. Moon's Equated Anomaly.

Add.

Degrees.	0 Signs.		1 Sign.		2 Signs.		3 Signs.		4 Signs.		5 Signs.		Degrees.
	H.	M.	H.	M.	H.	M.	H.	M.	H.	M.	H.	M.	
0	0	0	5	13	8	47	9	47	8	9	4	35	30
1	0	11	5	22	8	52	9	46	8	3	4	26	29
2	0	21	5	31	8	56	9	45	7	57	4	17	28
3	0	33	5	40	9	0	9	44	7	54	4	9	27
4	0	44	5	49	9	5	9	43	7	46	4	0	26
5	0	55	5	57	9	8	9	42	7	40	3	51	25
6	1	6	6	6	9	12	9	40	7	34	3	43	24
7	1	17	6	14	9	16	9	38	7	27	3	34	23
8	1	28	6	23	9	19	9	36	7	21	3	25	22
9	1	39	6	31	9	22	9	34	7	14	3	16	21
10	1	50	6	39	9	25	9	32	7	8	3	7	20
11	2	0	6	47	9	28	9	30	7	1	2	58	19
12	2	11	6	55	9	31	9	27	6	54	2	49	18
13	2	22	7	2	9	33	9	24	6	47	2	40	17
14	2	33	7	10	9	35	9	21	6	40	2	30	16
15	2	43	7	17	9	37	9	18	6	33	2	21	15
16	2	54	7	24	9	39	9	14	6	26	2	12	14
17	3	4	7	31	9	41	9	11	6	18	2	3	13
18	3	14	7	38	9	42	9	7	6	11	1	54	12
19	3	25	7	45	9	44	9	3	6	3	1	44	11
20	3	35	7	51	9	45	8	59	5	56	1	35	10
21	3	45	7	58	9	47	8	55	5	48	1	26	9
22	3	55	8	4	9	47	8	50	5	40	1	16	8
23	4	5	8	10	9	47	8	46	5	32	1	7	7
24	4	15	8	16	9	48	8	41	5	24	0	57	6
25	4	25	8	21	9	48	8	36	5	16	0	48	5
26	4	35	8	27	9	48	8	31	5	8	0	38	4
27	4	45	8	32	9	48	8	26	5	0	0	29	3
28	4	54	8	37	9	48	8	20	4	51	0	19	2
29	5	4	8	42	9	47	8	15	4	43	0	10	1
30	5	13	8	47	9	47	8	9	4	35	0	0	0
Deg.	11 Signs.		10 Signs.		9 Signs.		8 Signs.		7 Signs.		6 Signs.		Deg.

Subtract.

I. To

I. *To calculate the true time of New or Full Moon, in any given year and month of the 18th century.*

From Table I. (page 1, 2, 3) write out the mean time of New Moon in January, Old Stile, for the given year, with the mean Anomalies of the Sun and Moon, and the Sun's mean diſtance from the aſcending Node of the Moon's orbit. If you want the time of Full Moon, add the half lunation, with its Anomalies &c. at the foot of Table III. (page 5) to the foreſaid numbers, if the New Moon taken out falls before the 15th of January; but if it falls after, ſubtract the half lunation, &c. from the ſaid numbers; and write down the reſpective ſums or remainders. If you want to calculate for the New Stile, in any given year from *A. D.* 1752 to 1800, take out the Mean New Moon with its Anomalies, &c. from Table II. (page 4, 5)

In theſe additions, or ſubtractions, remember that 60 minutes make a degree,

degree, 30 degrees make a ſign, and 12 ſigns make a circle. And that, when the number of ſigns you ſubtract from is leſs than the number of ſigns to be ſubtracted, add 12 ſigns to the leſſer number; and then you will have a remainder to ſet down. *A ſign is marked thus* ^{s}, *a degree thus* $^{\circ}$, *and a minute thus* $'$.

When the required New or Full Moon is in any given month after January, add as many lunations from Table III. with their Anomalies, &c. to the numbers taken out for January, as the given month is after January; ſetting them in order below the January numbers: and theſe added together will give the Mean time of New or Full Moon, with the Anomalies thereto belonging, for the month deſired.

With the number of days added together, enter Table VI. (page 9) under the given month; and againſt that number you have the day of New or Full Moon in the left hand column (under

(under *Days*) which you are to ſet before the hours and minutes already found.

But, as will ſometimes happen, if the ſaid number of days falls ſhort of any in the column under the given month, add one lunation and its Anomalies to the foreſaid ſums; and with this new number of days enter Table VI. under the given month, where you are ſure to find it the ſecond time, if the firſt falls ſhort.

Then, with the ſigns and degrees of the Sun's mean Anomaly, enter Table VII. (page 10) and therewith take out the *firſt Equation from mean to true Syzygy*, making proportions in the Table for the minutes of Anomaly above whole degrees, becauſe the Tables give the Equations only to whole degrees. Subtract this Equation from the mean time of New or Full Moon, if the ſigns are at the head of the Table, in which caſe the degrees are in the left hand column and reckoned downward: but if the ſigns of Anomaly are at the foot of the Table, in

which

which caſe the degrees thereof are in the right hand column, and reckoned upward, add the Equation to the above found time of New or Full Moon.

With the ſigns and degrees of the Sun's mean Anomaly enter Table VIII, and therewith take out the *Equation of the Moon's mean Anomaly*; and apply that Equation to the Moon's mean Anomaly, ſubtracting it therefrom if the ſigns are at the head of the Table, and their degrees at the left hand; but adding it to the mean Anomaly of the Moon, if the ſigns of the Sun's Anomaly be at the foot of the Table, and their degrees at the right hand; and you will have the Moon's equated Anomaly; with which, enter Table IX. and take out the Equation anſwering thereto, adding it to the former equated time, if the ſigns are at the head of the Table, but ſubtracting it therefrom, if they are at the foot; and the reſult will give the true time of the required New or Full Moon, near enough for any common Almanack.

The

The Tables begin the day at Noon, and reckon the hours and minutes thence forward to the noon of the following day. They give the right time in all the months of common years, and in all the months after February in Leap years. But in January and February, in Leap years, a day muſt be added to the time given by the Tables.

Example I.

For the true time of Full Moon in March 1764, *New Stile.*

By the Precepts.	New Moon.	Sun's Anom.	Moon's Anom.	Sun from Node.
	D. H. M	S ° ′	S. ° ′	S. ° ′
To Jan. 1764	2 7 25	6 4 2	8 21 57	9 3 31
add ½ Lun.	14 18 22	0 14 33	6 12 54	0 15 20
2 Lunations	59 1 28	1 28 13	1 21 38	2 1 20
Full ☽ Mar.	17 3 15	8 16 48	4 26 29	11 20 11
Firſt Equ.	+4 6	Ar. 1 Eq.	+1 33	
Second Equ.	17 7 21		4 28 2	
	+4 50		Ar. 2 Eq.	
True time	17 12 11			

By this ſhort proceſs it appears, that the true time of the required Full Moon was the 17th of March, at 11 minutes paſt 12 o'clock at night. A few more examples will make the whole matter plain.

EXAMPLE II.

For the true time of New Moon in April 1764, *New Stile.*

By the Precepts.	New Moon.	Sun's Anom.	Moon's Anom.	Sun from Node.
	D. H. M.	S ° ′	S. ° ′	S. ° ′
Jan. 1764 +3 Lun.	2 7 25 88 14 12	6 4 2 2 27 19	8 21 57 2 17 27	9 3 31 3 2 1
March Firſt Equ.	31 21 37 +4 11	9 1 21 Ar. 1 Eq.	11 9 24 +1 35	0 5 32
Sec. Equ.	32 1 48 −3 25		11 10 59 Ar. 2 Eq.	
Tr. time	31 22 23			

This ſhews the true time to be at 22 hours, 23 minutes, after the noon of the 31ſt of March; which is the 1ſt of April at 23 minutes paſt X in the morning.

II. *To*

II. *To calculate the true time of New or Full Moon, in any given year and month, of any century, between the Chriſtian Æra and the 18th Century. Old Stile.*

In Table I. find a year in the 18th Century, of the ſame number with that in the Century propoſed, and take out the numbers belonging thereto as in the preceding Examples. Then, from Table IV. take out the numbers anſwering to the number of Centuries before the 18th, ſubtracting them from thoſe of the 18th, and ſetting down the remainder.

To this remainder join the numbers for as many Centuries, from Table V. ſubtracting thoſe for the New Moon, and Sun's diſtance from the Node, from the ſaid remainder, and adding thoſe for the Moon's Anomaly to it; and the reſult will give the mean time of New Moon in January, the year of the Century propoſed: which being found,

work, in all reſpects, for the true time of New or Full Moon in January or any other month of that year, as already ſhewn.

N. B. If the days annexed to the Centuries taken out from Table IV. exceed the number of days from the beginning of January, taken out in the 18th Century, add a Lunation and its Anomalies &c. from Table III. to thoſe taken out from the 18th Century; and then you can make a ſubtraction.

In all calculations for New or Full Moon, either before or after the 18th Century, the variation numbers anſwering to the Centuries in Table V. muſt be ſubtracted from the mean time of New Moon, and from the Sun's mean diſtance from the Node; and added to the Moon's mean Anomaly, as found for the given time, by the preceding Tables.

EXAMPLE

EXAMPLE III.

For the true time of New Moon in April, A. D. 237.

From *A. D.* 1737 ſubtract 15 Centuries (*viz.* 1500 years) and there will remain 237.

By the Precepts.	New Moon.	Sun's Anom.	Moon's Anom.	Sun from Node.
	D. H. M.	S. ° ′	S. ° ′	S. ° ′
Jan. 1737 —1500 Years	19 14 58 5 23 44	7 1 58 11 20 27	10 4 59 5 28 2	4 20 14 7 19 39
Remains Var. 1500 Y.	13 15 14 —1 34	7 11 31 — — —	4 6 57 +1 30	9 0 35 —0 30
Jan. A. D. 237 +3 Lun.	13 13 40 88 14 12	7 11 31 2 27 19	4 8 27 2 17 27	9 0 5 3 2 1
April. Firſt Equ.	12 3 52 +3 13	10 8 50 Ar. 1 Eq.	6 25 54 +1 13	0 2 6
Sec. Equ.	12 7 5 —4 10		6 27 7 Ar. 2 Eq.	
True time	12 2 55			

Hence, the true time required is April 12, at 55 minutes paſt II in the Afternoon.

II. *To*

III. *To calculate the true time of New or Full Moon in any given year and month before the Christian Æra; Old Stile.*

Find a year in the 18th Century, Old Stile, which being added to the given number of years before Chriſt, diminiſhed by one, ſhall make a compleat number of Centuries.

Find this number of Centuries in Table IV. and ſubtract the numbers belonging to them from thoſe for January, in the 18th Century; and to the remainders join the variations for the like number of Centuries from Table V. and then proceed, as above taught, in applying the Equations to gain the true time required.

The Moon's motion in her Orbit being now quicker than it was in former ages, is the reaſon for our giving the fifth Table, anſwering to her accelerations.

EXAMPLE IV.

For the true time of New Moon in May, Old Stile, the year before Christ 585.

The years 584, added to 1716, make 2300 years, or 20 compleat Centuries.

By the Precepts.	New Moon.	Sun's Anom.	Moon's Anom.	Sun from Node.
	D. H. M.	S. ° ′	S. ° ′	S. ° ′
Jan. 1716	11 16 6	6 24 35	2 12 38	2 25 54
−2300 Years	11 3 39	11 17 18	1 4 43	7 24 11
Remains	0 12 27	7 7 17	1 7 55	7 1 43
Var. 2300 Y.	−3 40	— — —	+3 32	−1 11
B. Chr. 585	0 8 47	7 7 17	1 11 27	7 0 32
+5 Lun.	147 15 40	4 25 32	4 9 5	5 3 21
May	28 0 27	0 2 49	5 20 32	0 3 53
First Equ.	−12	Ar. 1 Eq.	−5	
	28 0 15		5 20 27	
Sec. Equ.	+1 31		Ar. 2 Eq.	
Tr. time May	28 1 46			

So the true time was May 28th, at 46 minutes past 1 o'clock in the Afternoon.

IV. *To*

IV. *To calculate the true time of New or Full Moon in any year and month after the* 18*th Century, in the old Stile.*

Find a year of the ſame number in the 18th Century with that of the Century propoſed, and take out the New Moon and Anomalies for January, from Table I. for the ſaid year in the 18th Century: then from Table IV. take out the numbers for the Centuries after the 18th, adding them to thoſe of the 18th; to which join the Centurial variations, and then proceed for the true time of New or Full Moon as ſhewn in the former Precepts.

EXAMPLE

Example V.

For the true time of Full Moon in April, Old Stile, A. D. 1903.

To A. D. 1703 add 200 years, and the sum will be A. D. 1903.

By the Precepts.	New Moon.	Sun's Anom.	Moon's Anom.	Sun from Node.
	D. H. M.	S ° ′	S ° ′	S. ° ′
Jan. 1703 +½ Lun.	6 5 55 14 18 22	6 18 38 0 14 33	7 26 8 6 12 54	6 7 57 0 15 20
Full ☽ Jan. +200 Years	21 0 17 8 16 10	7 3 11 0 6 29	2 9 2 5 0 37	6 23 17 9 8 48
Var. 200 Years	29 16 27 −2	7 9 40 — — —	7 9 39 +2	4 2 5 −1
January +3 Lun.	29 16 25 88 14 12	7 9 40 2 27 19	7 9 41 2 17 27	4 2 4 3 2 1
April 1903 First Equ.	28 6 37 +3 18	10 6 59 Ar. 1 Eq.	9 27 8 +1 15	7 4 5
Second Equ.	28 9 55 −8 55		9 28 23 Ar. 2 Eq.	
Tr. time, Apr.	28 1 0			

Thus the true time is found to be April 28th, at 1 o'clock in the Afternoon.

In calculating forward from A. D. 1800, the eaſieſt way is to keep by the Old Stile, and then reduce it to the New, by adding the days difference of Stiles, which will be 12, from A. D. 1800 to 1900.

If the Old and New Stiles had exiſted from the beginning, there would have been no difference between them in A. D. 200. But from that time forward there would to the end of the world. And, in order to find always how many days (from the 200th year afer Chriſt's birth) muſt be added to the Old Stile to reduce it to the New, in any given Century, obſerve the following rule.

Divide the number of the given Century by 4, *and (without regarding the remainder, when there is any) add* 3 *to the quotient; then ſubtract the ſum from the number of the Century, and the remainder will be the number of days ſought.*

Thus, for the 18th Century: which began with A. D. 1701, and will end with

with A. D. 1800; the fourth part of 18 (omitting fractions) is 4, which added to 3 makes 7, and 7 being subtracted from 18, leaves 11 remaining, for the number of days between the Old and New Stile.

Again, for the 19th Century, which will begin with A. D. 1801, and end with A. D. 1900, a fourth part of 19 (without regarding fractions) is 4, which being added to 3 makes 7; and 7 being subtracted from 19, leaves 12 remaining for the number of days that muſt be added to the Old Stile to reduce it to the New, from A. D. 1800 to 1900; and ſo on.

When it appears, by ſuch as the foregoing calculations of New and Full Moons, that the Sun's diſtance from the [Aſcending] Node of the Moon's Orbit is leſs than 0 Signs 18 degrees, or more than 5 Signs 12 degrees, ſo as not to exceed 6 Signs 18 degrees; or when it is more than 11 Signs 12 degrees, at the time of New Moon, the Sun will be eclipſed at that time. And

 when

when the Sun's diſtance from the Node is leſs than 0 Signs 12 degrees, or any thing between 5 Signs 18 degrees, and 6 Signs 12 degrees; or more than 11 Signs 18 degrees, at the time of Full Moon, the Moon will be eclipſed at that time. On theſe principles it appears that there muſt be Eclipſes at the times mentioned in all the preceding Examples except the laſt. The reaſon of this is, that the Deſcending Node of the Moon's Orbit is directly oppoſite to the Aſcending; that is, they are juſt 6 Signs from each other. And when the Sun is within 18 degrees of either of the Nodes at the time of New Moon, the Sun will be eclipſed at that time. And when the Sun is within 12 degrees of either of the Nodes at the time of Full Moon, the Moon will then be eclipſed.

Becauſe moſt people are ſatisfied with knowing on what days of the months the Moon is New and Full, without regarding the time of the day, I ſhall here give a Table of all the

days of the months on which the mean changes of the Moon fall, from A. D. 1752 to 1800, in the New Stile. The days of Full Moons are then eaſily found; for when the Change happens before the 15th day of the month, 15 days added to the day of change, will give the day of Full Moon; and when the change is after the 15th day of the month, 15 days ſubtracted therefrom will give the day of Full Moon.

Within the above limits, the day of any month on which the Moon changeth, in any given year, is found under that month, and right againſt the year. Thus, ſuppoſe it was required to find on what day of March the change happens in A. D. 1767: under March at the head of the Table, and againſt 1767 at the left hand is 30; the day of the change required.

Where the figures are double, as $\frac{1}{30}$, or $\frac{1}{31}$, againſt any year, and under any month; they ſhew that the Moon changes on the 1ſt day of that month, and alſo on the 30th or 31ſt thereof.

A Table shewing on what days of the months the mean changes of the Moon fall, from A. D. 1752, *to* A. D. 1800. *New Stile.*

Years.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1752	16	14	15	13	13	12	11	10	8	8	6	6
1753	4	3	4	3	2	1	30	28	27	26	25	25
						31						
1754	23	22	23	22	21	20	19	18	16	16	14	14
1755	12	11	12	11	11	10	9	7	6	5	4	3
1756	1	—	1	29	28	27	26	25	24	23	22	21
	31		30									
1757	20	18	20	18	18	16	16	14	13	12	11	10
1758	9	8	9	8	7	6	5	4	2	2	30	29
										31		
1759	28	26	28	27	26	25	24	23	21	21	19	19
1760	17	16	16	15	14	13	12	11	9	9	7	7
1661	6	4	6	4	4	2	2	30	28	28	27	26
							31					
1762	25	23	25	23	23	21	21	19	18	17	16	15
1763	14	12	14	13	12	11	11	9	7	7	5	5
1764	3	2	2	1	30	28	28	27	25	25	23	23
				30								
1765	21	18	21	20	19	18	17	16	14	14	12	12
1766	11	9	11	9	9	7	7	5	4	3	2	1
												31
1767	29	28	30	28	28	26	26	24	23	22	21	20
1768	19	17	18	16	16	14	14	13	11	11	9	9
1769	7	6	7	6	5	4	3	2	30	29	28	27
								31				
1770	26	25	26	25	24	23	22	21	19	19	17	17
1771	15	14	16	14	14	12	12	10	9	8	7	6
1772	5	3	4	2	2	30	30	28	27	26	24	24
					31							
1773	23	21	23	21	21	19	19	17	16	16	14	14
1774	12	11	12	11	10	9	8	7	5	4	3	3
1775	1	—	2	30	29	28	27	26	24	24	22	22
	31		31									
1776	20	19	19	18	17	16	16	14	13	12	11	10

The

The Table concluded. New Stile.

Years.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1777	9	7	9	7	7	5	5	3	2	1	30	29
										31		
1778	28	26	28	26	26	24	24	22	21	20	19	18
1779	17	16	17	16	15	14	13	12	10	10	8	8
1780	6	4	5	4	3	2	2	30	28	28	26	26
							31					
1781	24	23	24	23	22	21	20	19	17	17	16	15
1782	14	12	14	12	12	10	10	8	7	6	5	4
1783	3	2	3	2	1	29	29	27	26	25	24	23
					31							
1784	22	20	21	19	19	18	17	16	14	14	12	12
1785	10	9	10	9	8	7	6	5	3	3	2	2
												31
1786	29	28	29	28	27	26	25	24	22	22	20	20
1787	18	17	19	17	17	15	15	13	12	11	10	9
1788	8	6	7	5	5	4	3	2	30	29	28	27
								31				
1789	26	24	26	24	24	22	22	21	19	19	17	17
1790	15	14	15	14	13	12	11	10	8	8	6	6
1791	5	3	5	3	3	1	1	29	27	27	25	25
							30					
1792	23	22	22	21	21	19	19	17	16	15	14	13
1793	12	10	12	10	10	8	8	6	5	5	3	2
1794	1	—	1	29	29	27	27	25	24	23	22	22
	31		31									
1795	20	19	20	19	18	17	16	15	13	13	11	11
1796	9	8	8	7	7	5	5	3	2	1	29	29
										31		
1797	27	26	27	26	25	24	23	22	21	20	19	18
1798	17	15	17	15	15	13	13	11	10	9	8	8
1799	6	5	6	5	4	3	2	1	29	28	27	26
								30				
1800	25	23	25	23	22	21	20	19	17	17	15	15

This Table begins the day at midnight, which is according to the common way of reckoning.

Look

Look for the given year in the left hand column, and againſt it under the given month you have the day of mean New Moon in that month.

Of the cauſes and times of Eclipſes.

An Eclipſe of the Sun is cauſed by the Moon's opaque body paſſing between the Sun and thoſe parts of the earth from which ſhe hides the whole or part of the Sun: and this can never happen but at the time of New Moon.

An Eclipſe of the Moon is cauſed by the whole or part of her body paſſing through the earth's ſhadow: which can never happen but when the Moon is full.

If the Moon's Orbit lay in the plane of the Ecliptic (in which the earth always moves, and the Sun appears to move) the Sun would be eclipſed at the time of every New Moon; and the Moon would be eclipſed at the time of every Full.

But

But one half of the Moon's Orbit lies on the North side of the Ecliptic, and the other half on the South side of it. Therefore, the Moon's Orbit intersects the Ecliptic only in two opposite points, which are called the *Moon's Nodes*; and the angle which the Moon's Orbit makes with the Ecliptic is 5° 18′. The intersection from which the Moon ascends Northward from the Ecliptic is called the Moon's *Ascending Node*; and the opposite intersection, from which the Moon descends Southward from the Ecliptic is called the Moon's *Descending Node*. These Nodes move backward in the Ecliptic $19\frac{1}{3}$ degrees every year, from the consequent toward the antecedent signs, and therefore they go quite round the Ecliptic, in 18 years, 223 days, and 5 hours.

From the time of the Sun's being in conjunction with either of the Moon's Nodes, to the time of his being in conjunction with the other, is about $173\frac{1}{2}$ days, at a mean rate; within which

number of days the Eclipses must always happen, in different times of the year. *The days of these Conjunctions are shewn in the following Table, from A. D.* 1752 *to* 1800, N. S.

Mean Conjunctions of the Sun and Nodes.					
Years.	Asc. Node. Mon. D.	Desc. Node. Mon. D.	Years.	Asc. Node. Mon. D.	Desc. Node. Mon. D.
1752	Nov. 5	May 16	1777	July 10	Jan. 17
1753	Oct. 18	Apr. 28			Dec. 30
1754	Sept. 29	Apr. 9	1778	July 21	Dec. 11
1755	Sept. 11	Mar. 21	1779	June 2	Nov. 23
1756	Aug. 23	Mar. 2	1780	May 14	Nov. 4
1757	Aug. 5	Feb. 12	1781	Apr. 26	Oct. 16
1758	July 17	Jan. 25	1782	Apr. 8	Sept. 28
1759	June 29	Jan. 6	1783	Mar. 20	Sept. 10
		Dec. 18	1784	Mar. 1	Aug. 22
1760	June 10	Nov. 30	1785	Feb. 10	Aug. 1
1761	May 22	Nov. 11	1786	Jan. 22	July 15
1762	May 4	Oct. 24	1787	Jan. 4	June 27
1763	Apr. 15	Oct. 6		Dec. 16	
1764	Mar. 27	Sept. 17	1788	Nov. 27	June 9
1765	Mar. 8	Aug. 29	1789	Nov. 8	May 20
1766	Feb. 18	Aug. 10	1790	Oct. 21	May 2
1767	Jan. 31	July 23	1791	Oct. 3	Apr. 13
1768	Jan. 12	July 4	1792	Sept. 14	Mar. 25
	Dec. 23		1793	Aug. 27	Mar. 7
1769	Dec. 5	June 15	1794	Aug. 8	Feb. 16
1770	Nov. 17	May 28	1795	July 21	Jan. 28
1771	Oct. 29	May 9	1796	July 2	Jan. 9
1772	Oct. 10	Apr. 20			Dee. 22
1773	Sept. 22	Apr. 2	1797	June 14	Dec. 4
1774	Sept. 4	Mar. 14	1798	May 26	Nov. 15
1775	Aug. 16	Feb. 23	1799	May 8	Oct. 28
1776	July 28	Feb. 4	1800	Apr. 19	Oct. 9

When

When the Moon changes within 18 days before or after the day of the Sun's being in conjunction with either of her Nodes, the Sun will be eclipſed: and when the Moon is full within 12 days before or after the day of the Sun's conjunction with either of the Nodes, the Moon will be eclipſed. At greater diſtances of the Sun from the Nodes, there can be no Eclipſes of theſe Luminaries.

As the Table contained on page 30 and 31 ſhews the days on which the mean changes of the Moon happen, and the Moon is always full on the 15th day before or after the change; and the Table on page 34 ſhews the days on which the Sun is in conjunction with the Moon's Nodes; we may eaſily find by theſe Tables on what days of any given year from A. D. 1752 to 1800, the Sun and Moon muſt be eclipſed. As, for example.

In the year 1766, the Sun is in conjunction with the Moon's Aſcending Node on the 18th of February, and

with the Deſcending Node on the 10th of Auguſt. Now, I find by the Table, page 30, and 31, that in the year 1766, the changes of the Moon are on Jan. 11, Feb. 9, March 11, April 9, May 9, June 7, July 7, Aug. 5, Sept. 4, Oct. 3, Nov. 2, Dec. 1 and 31; and conſequently, as the change on Feb. 9th is within 18 days of Feb. 18th when the Sun is in conjunction with the Aſcending Node, the Sun muſt be eclipſed at the time of that change. And as the change on Auguſt 5 is within 18 days of Auguſt 10, when the Sun is in conjunction with the Deſcending Node of the Moon's Orbit, the Sun muſt be eclipſed at the time of that change alſo. But as all the other changes of the Moon in that year are more than 18 days from the times of the conjunctions of the Sun and Nodes, there can be no more than the two abovementioned Eclipſes of the Sun in the year 1766.

By adding 15 days to all the changes of the Moon, in the ſame year, we

we find the days of all the Full Moons to be Jan. 26, Feb. 24, March 26, April 24, May 24, June 22, July 22, Auguſt 20, Sept. 19, Oct. 18, Nov. 17, and Dec. 16. But of all theſe Full Moons, there are only two which happen within 12 days of the conjunctions of the Sun and Nodes; *viz.* thoſe on the 24th of February and 20th of Auguſt: and therefore, it is only on theſe two days of the year 1766, that the Moon can be eclipſed.

And thus we have a very plain and eaſy method for finding how many Eclipſes there muſt be of the Sun and Moon in any given year, and the days on which they muſt fall, according to the mean times of New and Full Moons, from *A. D.* 1752 to *A. D.* 1800. But to ſhew how to calculate the true times and places of Eclipſes for different parts of the Earth, would ſwell out this volume far beyond the intended bulk: and therefore, for ſuch calculations and projections, I beg leave to refer the curious reader to my ſyſtem

of

of Aſtronomy, printed for Mr. Millar, Bookſeller in the Strand, London; to be now had of Mr. Cadell, ſucceſſor to Mr. Millar, at his ſhop oppoſite Catherine ſtreet in the Strand.

The following Table ſhews the Sun's true place in the Ecliptic, and his declination from the Equator, at the noon of every day of the ſecond year after Leap year, on the meridian of Greenwich. The ſigns of the Ecliptic are marked in the Table as follows.

Aries ♈, *Taurus* ♉, *Gemini* ♊, *Cancer* ♋, *Leo* ♌, *Virgo* ♍, *Libra* ♎, *Scorpio* ♏, *Sagittarius* ♐, *Capricornus* ♑, *Aquarius* ♒, and *Piſces* ♓.

A Table of the Sun's Declination is very uſeful for finding the Latitudes of places on the Earth. And as the method of doing this by the Declination of the Sun is generally known, we have given the following Table for that purpoſe, to the neareſt mean between Leap year, and the firſt, ſecond, and third year after.

A Table

A Table shewing the Sun's Place and Declination.

Days	January. Sun's Pl. S.	°	′	S.'s Dec. °	′	February. Sun's Pl. S.	°	′	S.'s Dec. °	′
1	♑	11	5	S. 23	1	♒	12	38	S. 17	3
2		12	6	22	56		13	39	16	45
3		13	8	22	50		14	40	16	28
4		14	9	22	44		15	41	16	10
5		15	10	22	37		16	41	15	52
6		16	11	22	30		17	42	15	33
7		17	12	22	22		18	43	15	15
8		18	13	22	14		19	43	14	56
9		19	14	22	6		20	44	14	36
10		20	16	21	57		21	45	14	17
11		21	17	21	48		22	45	13	57
12		22	18	21	38		23	46	13	37
13		23	19	21	28		24	46	13	17
14		24	20	21	17		25	47	12	57
15		25	21	21	6		26	47	12	37
16		26	22	20	55		27	48	12	16
17		27	24	20	43		28	48	11	55
18		28	25	20	31		29	48	11	34
19		29	26	20	18	♓	0	49	11	12
20	♒	0	27	20	5		1	49	10	51
21		1	28	19	52		2	50	10	29
22		2	29	19	39		3	50	10	7
23		3	30	19	24		4	50	9	45
24		4	31	19	11		5	51	9	23
25		5	32	18	55		6	51	9	1
26		6	33	18	40		7	51	9	38
27		7	34	18	25		8	51	8	16
28		8	35	18	9		9	51	8	53
29		9	35	17	53					
30		10	36	17	37					
31		11	37	17	20					

N. signifies North Decl. and S. South Decl.

To find the Sun's Place and Declination at the Noon of any given day, in the second year after Leap year;

Look

The Table continued.

Days	March. Sun's Pl. S. °	′	S.'s Dec. °	′	April. Sun's Pl. S. °	′	S.'s Dec. °	′
1	♓ 10	52	S. 7	30	♈ 11	38	N. 4	37
2	11	52	7	7	12	37	5	0
3	12	52	6	45	13	36	5	23
4	13	52	6	21	14	35	5	45
5	14	52	5	58	15	34	6	8
6	15	52	5	35	16	33	6	31
7	16	51	5	12	17	31	6	53
8	17	51	4	49	18	30	7	16
9	18	51	4	25	19	29	7	38
10	19	51	4	2	20	28	8	1
11	20	51	3	38	21	27	8	23
12	21	50	3	15	22	25	8	44
13	22	50	2	51	23	24	9	6
14	23	50	2	27	24	23	9	28
15	24	49	2	4	25	21	9	49
16	25	49	1	40	26	20	10	11
17	26	49	1	16	27	18	10	32
18	27	48	0	53	28	17	10	53
19	28	48	0	29	29	15	11	14
20	29	47	0	5	♉ 0	14	11	34
21	♈ 0	47	N. 0	19	1	12	11	55
22	1	46	0	42	2	11	12	15
23	2	46	1	6	3	9	12	35
24	3	45	1	30	4	7	12	55
25	4	44	1	53	5	6	13	15
26	5	43	2	16	6	4	13	34
27	6	43	2	40	7	2	13	54
28	7	42	3	4	8	0	14	12
29	8	41	3	27	8	59	14	31
30	9	40	3	50	9	57	14	50
31	10	39	4	13				

Look for the month at the top of the Table, and under it againſt the given day of the month, you have the Sun's

The Table continued.

Days	May. Sun's Pl.			May. S's Dec.		June. Sun's Pl.			June. S's Dec.	
	S.	°	′	°	′	S.	°	′	°	′
1	♉	01	55	N. 15	8	♊	10	44	N. 22	6
2		11	53	15	26		11	41	22	14
3		12	51	15	43		12	39	22	21
4		13	49	16	1		13	36	22	28
5		14	47	16	18		14	34	22	35
6		15	45	16	35		15	31	22	42
7		16	43	16	52		16	28	22	47
8		17	41	17	8		17	26	22	53
9		18	39	17	24		18	23	22	58
10		19	36	17	40		19	20	23	3
11		20	34	17	55		20	18	23	8
12		21	32	18	11		21	15	23	12
13		22	30	18	26		22	12	23	15
14		23	28	18	40		23	9	23	18
15		24	25	18	55		24	7	23	21
16		25	23	19	9		25	4	23	23
17		26	21	19	22		26	1	23	25
18		27	19	19	36		26	58	23	26
19		28	16	19	49		27	56	23	28
20		29	14	20	1		28	53	23	29
21	♊	0	11	20	14		29	50	33	29
22		1	9	20	26	♋	0	47	23	29
23		2	7	20	37		1	45	23	28
24		3	4	20	49		2	42	23	27
25		4	2	21	0		3	39	23	26
26		4	59	21	10		4	36	23	24
27		5	57	21	20		5	33	23	22
28		6	54	21	30		6	31	23	19
29		7	52	21	40		7	28	23	16
30		8	49	21	49		8	25	23	13
31		9	47	21	58					

Sun's Place in the Ecliptic, and his Declination as it is then North or South.

By means of this Table, and the Table of the Semidiurnal Arcs of the

 Sun

The Table continued.

Days	July.					August.				
	Sun's Pl.			S's Dec.		Sun's Pl.			S's Dec.	
	S.	°	′	°	′	S.	°	′	°	′
1	♋	9	22	N.23	9	♌	8	58	N.18	3
2		10	19	23	5		9	55	17	48
3		11	16	23	0		10	53	17	32
4		12	14	22	55		11	50	17	16
5		13	11	22	50		12	48	17	0
6		14	8	22	44		13	45	16	44
7		15	5	22	38		14	43	16	27
8		16	1	22	31		15	41	16	10
9		16	59	22	24		16	38	15	53
10		17	56	22	17		17	36	15	35
11		18	54	22	9		18	33	15	18
12		19	51	22	1		19	31	15	0
13		20	49	21	52		20	29	14	41
14		21	46	21	43		21	26	14	23
15		22	43	21	34		22	24	14	4
16		23	40	21	24		23	22	13	45
17		24	38	21	14		24	20	13	26
18		25	35	21	4		25	17	13	7
19		26	32	20	53		26	15	12	47
20		27	29	20	42		27	13	12	28
21		28	27	20	30		28	11	12	8
22		29	24	20	19		29	9	11	47
23	♌	0	21	20	7	♍	0	7	11	27
24		1	19	19	54		1	5	11	7
25		2	16	19	41		2	3	10	46
26		3	13	19	28		3	1	10	25
27		4	11	19	15		3	59	10	4
28		5	8	19	1		4	57	9	43
29		6	6	18	47		5	55	9	22
30		7	3	18	33		6	53	9	0
31		8	0	18	18		7	51	8	38

Sun and Moon, the times of riſing and ſetting of the Sun, on any day of the year, may be found, in all latitudes from 48 degrees to 59 incluſive. The

Sun's

The Table continued.

Days	September.						October.					
	Sun's Pl.			S's Dec.			Sun's Pl.			S's Dec.		
	S.	°	′		°	′	S.	°	′		°	′
1	♍	8	49	N.	8	17	♎	8	8	S.	3	14
2		9	47		7	55		9	7		3	37
3		10	46		7	32		10	7		4	1
4		11	44		7	10		11	6		4	24
5		12	42		6	48		12	5		4	47
6		13	40		6	26		13	4		5	10
7		14	39		6	3		14	4		5	33
8		15	37		5	41		15	3		5	56
9		16	35		5	18		16	3		6	20
10		17	34		4	55		17	2		6	42
11		18	32		4	32		28	1		7	5
12		19	31		4	9		19	1		7	28
13		20	29		3	47		20	0		7	50
14		21	28		3	23		21	0		8	13
15		22	26		3	0		22	0		8	35
16		23	25		2	37		23	0		8	57
17		24	24		2	14		23	59		9	19
18		25	22		1	51		24	59		9	41
19		26	21		1	27		25	59		10	3
20		27	20		1	4		26	58		10	25
21		28	19		0	40		27	58		10	46
22		29	17		0	17		28	58		11	8
23	♎	0	16	S.	0	7		29	58		11	29
24		1	15		0	30	♏	0	58		11	50
25		2	14		0	53		1	58		12	11
26		3	13		1	17		2	58		12	31
27		4	12		1	40		3	58		12	52
28		5	11		2	4		4	58		13	12
29		6	10		2	27		5	58		13	32
30		7	9		2	51		6	58		13	52
31								7	58		14	11

Sun's Declination is alſo uſeful for finding the Latitudes of Places; for which I have given a great variety of Rules in my Book of Lectures on Mechanics,

 Hydro-

The Table concluded.

Days.	November. Sun's Pl.			S's Dec.		December. Sun's Pl.			S's Dec.	
	S.	°	′	°	′	S.	°	′	°	′
1	♏	8	58	S. 14	31	♐	9	16	S. 21	53
2		9	58	14	50		10	17	22	2
3		10	59	15	9		11	18	22	11
4		11	59	15	28		12	19	22	19
5		12	59	15	46		13	20	22	27
6		13	59	16	4		14	22	22	34
7		15	0	16	22		15	23	22	41
8		16	0	16	39		16	24	22	47
9		17	0	16	57		17	25	22	53
10		18	1	17	14		18	26	22	59
11		19	1	17	30		19	27	23	4
12		20	2	17	47		20	28	23	8
13		21	2	18	3		21	29	23	12
14		22	3	18	19		22	30	23	16
15		23	4	18	34		23	31	23	19
16		24	4	18	49		24	32	23	22
17		25	5	19	4		25	33	23	25
18		26	5	19	19		26	34	23	21
19		27	6	19	33		27	35	23	28
20		28	7	19	47		28	36	23	29
21		29	7	20	0		29	37	23	29
22	♐	0	8	20	13	♑	0	38	23	29
23		1	9	20	26		1	39	23	28
24		2	10	20	38		2	40	23	27
25		3	11	20	50		3	42	23	26
26		4	11	21	1		4	43	23	24
27		5	12	21	12		5	44	23	21
28		6	13	21	23		6	45	23	19
29		7	14	21	33		7	46	23	15
30		8	15	21	43		8	48	23	11
31							9	49	23	7

Hydroſtatics, Pneumatics, and Optics; printed for Mr. Millar, Bookſeller in the Strand, London.

A Table

A Table of Semi-diurnal Arcs, for ſhewing the times of riſing and ſetting of the Sun and Moon.

North Declination of the Sun and Moon. Declin.	Latitude of the Place. 48°		49°		50°	
	Sun	Moon	Sun	Moon	Sun	Moon
Deg.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	6 8	6 20	6 8	6 20	6 8	6 20
2	6 12	6 24	6 12	6 24	6 13	6 24
3	6 17	6 29	6 17	6 29	6 18	6 29
4	6 21	6 33	6 21	6 33	6 22	6 34
5	6 25	6 38	6 26	6 38	6 27	6 39
6	6 30	6 43	6 31	6 44	6 32	6 45
7	6 34	6 48	6 36	6 49	6 37	6 50
8	6 39	6 53	6 41	6 54	6 42	6 55
9	6 44	6 57	6 45	6 58	6 47	7 1
10	6 48	7 1	6 50	7 3	6 52	7 7
11	6 53	7 6	6 55	7 9	6 57	7 12
12	6 58	7 12	7 0	7 14	7 2	7 17
13	7 3	7 17	7 5	7 19	7 7	7 21
14	7 8	7 22	7 10	7 24	7 11	7 29
15	7 13	7 27	7 15	7 31	7 18	7 33
16	7 18	7 32	7 21	7 36	7 24	7 39
17	7 23	7 37	7 26	7 41	7 29	7 44
18	7 28	7 43	7 31	7 46	7 35	7 50
19	7 34	7 49	7 37	7 52	7 41	7 56
20	7 39	7 54	7 43	7 58	7 47	8 2
21	7 45	8 0	7 49	8 5	7 53	8 9
22	7 50	8 6	7 55	8 11	7 59	8 15
23	7 56	8 12	8 1	8 17	8 6	8 22
½	7 59	8 15	8 4	8 20	8 9	8 25

To find the time of Sun-riſing and Sun-ſetting, on any given day of the year, in

The Table of Semi-diurnal Arcs continued.

South Declination of the Sun and Moon.

Declin.	Latitude of the Place.					
	48°		49°		50°	
	Sun	Moon	Sun	Moon	Sun	Moon
Deg.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	5 59	6 11	5 59	6 11	5 59	6 11
2	5 54	6 6	5 54	6 6	5 54	6 6
3	5 50	6 1	5 49	6 1	5 49	6 1
4	5 45	5 57	5 45	5 57	5 44	5 56
5	5 41	5 53	5 40	5 52	5 39	5 51
6	5 36	5 48	5 35	5 47	5 35	5 46
7	5 32	5 44	5 31	5 43	5 30	5 42
8	5 27	5 38	5 26	5 37	5 25	5 36
9	5 23	5 33	5 21	5 31	5 20	5 30
10	5 18	5 28	5 17	5 27	5 15	5 25
11	5 13	5 23	5 12	5 22	5 10	5 20
12	5 9	5 19	5 7	5 17	5 5	5 15
13	5 4	5 14	5 2	5 12	5 0	5 10
14	4 59	5 9	4 57	5 7	4 54	5 4
15	4 54	5 4	4 52	5 2	4 49	4 59
16	4 49	4 58	4 46	4 56	4 45	4 54
17	4 44	4 53	4 41	4 51	4 38	4 48
18	4 39	4 48	4 36	4 46	4 33	4 42
19	4 34	4 43	4 30	4 40	4 27	4 36
20	4 28	4 37	4 25	4 33	4 21	4 29
21	4 23	4 31	4 19	4 27	4 15	4 23
22	4 17	4 25	4 13	4 21	4 9	4 17
23	4 11	4 19	4 7	4 15	4 3	4 11
½	4 8	4 16	4 4	4 12	4 0	4 8

in all Latitudes from 48 *to* 59 *inclusive.*

Find the Sun's declination for the given day, by the preceding Table; then,

The of Table Semi-diurnal Arcs continued.

North Declination of the Sun and Moon. Declin.	Latitude of the Place. 51° Sun	51° Moon	52° Sun	52° Moon	53° Sun	53° Moon
Deg.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	6 8	6 20	6 9	6 21	6 9	6 21
2	6 13	6 25	6 14	6 26	6 14	6 26
3	6 18	6 30	6 19	6 31	6 19	6 31
4	6 22	6 34	6 24	6 36	6 25	6 37
5	6 27	6 39	6 29	6 41	6 30	6 42
6	6 33	6 46	6 34	6 47	6 36	6 49
7	6 38	6 51	6 40	6 53	6 41	6 54
8	6 43	6 56	6 45	6 58	6 47	6 59
9	6 48	7 1	6 50	7 3	6 52	7 5
10	6 54	7 7	6 56	7 9	6 58	7 11
11	6 59	7 13	7 1	7 15	7 3	7 17
12	7 4	7 18	7 7	7 21	7 9	7 23
13	7 10	7 24	7 12	7 26	7 15	7 29
14	7 15	7 29	7 18	7 32	7 21	7 35
15	7 21	7 35	7 23	7 33	7 27	7 41
16	7 27	7 41	7 30	7 44	7 33	7 47
17	7 33	7 48	7 36	7 51	7 40	7 54
18	7 38	7 54	7 42	7 58	7 46	8 1
19	7 45	8 1	7 49	8 5	7 53	8 9
20	7 51	8 7	7 55	8 11	8 0	8 16
21	7 57	8 13	8 2	8 18	8 7	8 23
22	8 4	8 20	8 9	8 25	8 14	8 30
23	8 11	8 27	8 16	8 32	8 22	8 37
½	8 15	8 31	8 20	8 36	8 26	8 41

then, in the Table of Semi-diurnal Arcs, under the Latitude of the place, and againſt the degrees of the Sun's Declination at the left hand (as the Declination is then North or South) you

The Table of Semi-diurnal Arcs continued.

South Declination of the Sun and Moon.

Declin.	Latitude of the Place. 51°		52°		53°	
	Sun	Moon	Sun	Moon	Sun	Moon
Deg.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	5 58	6 9	5 58	6 9	5 58	6 9
2	5 53	6 4	5 53	6 4	5 53	6 4
3	5 49	6 0	5 48	5 59	5 48	5 59
4	5 44	5 55	5 43	5 54	5 42	5 53
5	5 39	5 50	5 38	5 49	5 37	5 48
6	5 34	5 45	5 33	5 44	5 31	5 42
7	5 29	5 40	5 27	5 38	5 26	5 37
8	5 23	5 33	5 22	5 32	5 21	5 31
9	5 18	5 28	5 17	5 27	5 16	5 26
10	5 13	5 23	5 11	5 22	5 10	5 21
11	5 8	5 18	5 6	5 16	5 4	5 14
12	5 3	5 13	5 0	5 10	4 58	5 8
13	4 57	5 7	4 55	5 5	4 52	5 3
14	4 52	5 2	4 49	4 59	4 47	4 57
15	4 41	4 56	4 44	4 54	4 41	4 51
16	4 46	4 51	4 38	4 47	4 34	4 44
17	4 35	4 45	4 32	4 41	4 28	4 38
18	4 29	4 39	4 26	4 34	4 22	4 31
19	4 23	4 32	4 19	4 28	4 15	4 24
20	4 17	4 26	4 13	4 21	4 9	4 17
21	4 11	4 19	4 6	4 14	4 2	4 10
22	4 4	4 12	4 0	4 8	3 55	4 3
23	3 58	4 2	3 53	4 1	3 47	3 57
½	3 55	3 59	3 49	3 57	3 43	3 54

you have the Sun's Semi-diurnal Arc, or time of his ſetting, on that day; which Arc being doubled, gives the whole length of the day; and being ſubtracted from 12, gives the time of Sun riſing. Thus, ſuppoſe the Latitude

The Table of Semi-diurnal Arcs continued.

North Declination of the Sun and Moon.

Declin.	Latitude of the Place.					
	54°		55°		56°	
	Sun	Moon	Sun	Moon	Sun	Moon
Deg.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	6 9	6 21	6 9	6 21	6 10	6 22
2	6 15	6 26	6 15	6 27	6 16	6 28
3	6 20	6 32	6 21	6 33	6 22	6 34
4	6 26	6 38	6 27	6 39	6 28	6 40
5	6 31	6 44	6 32	6 45	6 34	6 47
6	6 37	6 51	6 38	6 52	6 40	6 54
7	6 43	6 57	6 44	6 58	6 46	7 0
8	6 48	7 2	6 50	7 4	6 52	7 6
9	6 54	7 8	6 56	7 10	6 58	7 12
10	7 0	7 14	7 2	7 16	7 5	7 19
11	7 6	7 20	7 8	7 22	7 11	7 25
12	7 12	7 26	7 15	7 29	7 18	7 32
13	7 18	7 32	7 21	7 35	7 24	7 38
14	7 24	7 37	7 28	7 42	7 31	7 46
15	7 31	7 45	7 34	7 49	7 39	7 53
16	7 37	7 52	7 41	7 56	7 45	8 0
17	7 44	7 59	7 48	8 3	7 52	8 7
18	7 51	8 7	7 55	8 10	8 0	8 15
19	7 58	8 14	8 2	8 18	8 7	8 23
20	8 5	8 21	8 10	8 26	8 15	8 31
21	8 12	8 28	8 18	8 34	8 24	8 40
22	8 20	8 36	8 26	8 42	8 32	8 48
23	8 28	8 44	8 34	8 50	8 41	8 57
½	8 32	8 49	8 38	8 54	8 46	9 1

tude to be 52 degrees North; and the day to be the 4th of May, when the Sun's Declination is 16 degrees North. Then, under 52° at the head of this Table, and againſt 16 degrees of North

 decli-

The of Table Semi-diurnal Arcs continued.

South Declination of the Sun and Moon.

Declin.	Latitude of the Place. 54°		55°		56°	
	Sun	Moon	Sun	Moon	Sun	Moon
Deg.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	5 58	6 10	5 58	6 10	5 58	6 10
2	5 53	6 5	5 53	6 5	5 52	6 4
3	5 47	5 59	5 47	5 59	5 46	5 58
4	5 42	5 54	5 41	5 53	5 40	5 52
5	5 36	5 47	5 35	5 46	5 34	5 45
6	5 30	5 41	5 29	5 40	5 28	5 39
7	5 25	5 36	5 23	5 34	5 22	5 33
8	5 19	5 30	5 17	5 28	5 16	5 27
9	5 13	5 23	5 12	5 22	5 10	5 20
10	5 8	5 18	5 6	5 17	5 3	5 14
11	5 2	5 12	4 59	5 10	4 57	5 8
12	4 56	5 6	4 53	5 3	4 51	5 1
13	4 50	5 0	4 47	4 56	4 44	4 53
14	4 44	4 55	4 41	4 50	4 37	4 46
15	4 37	4 48	4 34	4 43	4 31	4 40
16	4 31	4 41	4 27	4 36	4 24	4 33
17	4 23	4 34	4 21	4 29	4 17	4 25
18	4 18	4 26	4 14	4 22	4 9	4 17
19	4 11	4 19	4 7	4 15	4 2	4 10
20	4 4	4 12	3 59	4 7	3 54	4 2
21	3 57	4 5	3 52	3 59	3 46	3 54
22	3 50	3 58	3 44	3 52	3 38	3 45
23	3 42	3 49	3 36	3 44	3 29	3 36
½	3 38	3 44	3 32	3 40	3 25	3 32

declination, I find 7 hours 30 minutes to be the Sun's ſemi-diurnal arc on that day; which being doubled gives 15 hours for the whole length of the day. The ſaid arc ſhews that the Sun ſets

The Table of Semi-diurnal Arcs continued.

North Declination of the Sun and Moon. Declin.	Latitude of the Place. 57° Sun	57° Moon	58° Sun	58° Moon	59° Sun	59° Moon
Deg.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	6 10	6 22	6 10	6 22	6 11	6 23
2	6 16	6 28	6 17	6 29	6 17	6 30
3	6 22	6 34	6 23	6 35	6 24	6 37
4	6 29	6 41	6 30	6 42	6 31	6 44
5	6 35	6 48	6 36	6 49	6 38	6 51
6	6 41	6 55	6 43	6 56	6 44	6 58
7	6 48	7 2	6 49	7 3	6 51	7 5
8	6 54	7 8	6 56	7 9	6 58	7 12
9	7 1	7 15	7 3	7 16	7 5	7 19
10	7 7	7 21	7 10	7 24	7 13	7 27
11	7 14	7 28	7 17	7 31	7 20	7 34
12	7 21	7 35	7 24	7 39	7 27	7 42
13	7 28	7 42	7 31	7 46	7 35	7 50
14	7 35	7 50	7 39	7 54	7 43	7 58
15	7 42	7 57	7 46	8 2	7 51	8 6
16	7 49	8 4	7 54	8 10	7 59	8 15
17	7 57	8 12	8 2	8 18	8 7	8 23
18	8 5	8 21	8 10	8 26	8 16	8 32
19	8 13	8 29	8 19	8 35	8 25	8 42
20	8 21	8 38	8 28	8 45	8 35	8 52
21	8 30	8 47	8 37	8 55	8 45	9 2
22	8 39	8 56	8 47	8 5	8 55	9 13
23	8 49	9 6	8 57	9 15	9 6	9 24
½	8 54	9 11	9 2	9 20	9 11	9 29

ſets at 30 minutes after 7; and being ſubtracted from 12, leaves 4 hours 30 minutes, which ſhews that the Sun riſes at 30 minutes after 4 o'clock. In

The Table of Semi-diurnal Arcs, concluded.

South Declination of the Sun and Moon.

Declin.	Latitude of the Place.					
	57°		58°		59°	
	Sun	Moon	Sun	Moon	Sun	Moon
Deg.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	5 58	6 10	5 58	6 10	5 57	6 9
2	5 52	6 3	5 51	6 3	5 51	6 2
3	5 45	5 56	5 45	5 57	5 44	5 55
4	5 39	5 50	5 38	5 50	5 37	5 48
5	5 33	5 44	5 32	5 43	5 31	5 42
6	5 27	5 37	5 25	5 36	5 24	5 35
7	5 20	5 30	5 19	5 29	5 17	5 28
8	5 14	5 24	5 12	5 22	5 10	5 20
9	5 8	5 18	5 5	5 14	5 3	5 13
10	5 1	5 11	4 59	5 9	4 56	5 6
11	4 54	5 4	4 52	5 2	4 49	4 59
12	4 48	4 57	4 45	4 55	4 42	4 52
13	4 41	4 50	4 38	4 47	4 34	4 44
14	4 34	4 43	4 30	4 39	4 27	4 37
15	4 27	4 36	4 23	4 31	4 19	4 30
16	4 20	4 29	4 15	4 23	4 11	4 20
17	4 12	4 21	4 8	4 16	4 3	4 11
18	4 5	4 13	4 0	4 8	3 54	4 2
19	3 56	4 4	3 51	3 59	3 45	3 54
20	3 49	3 56	3 43	3 51	3 36	3 44
21	3 40	3 48	3 34	3 43	3 27	3 34
22	3 31	3 39	3 24	3 33	3 17	3 23
23	3 23	3 29	3 15	3 22	3 6	3 12
½	3 18	3 24	3 10	3 17	3 1	3 7

this Table, the effect of the refractions is taken into the account.

And, in any intermediate part of a degree of latitude and declination, it is easy to make proportions in the Table,

ble, for the rising and setting of the Sun.

To find the time of the Moon's rising and setting on any given day of the year.

This is a more operose problem than the former one for finding the rising and setting of the Sun. For here we must find the Moon's age, declination, and time of coming to the Meridian, on the given day.

The number of days elapsed between the last change and the given day, in any current Lunation, are the days of the Moon's age in that Lunation. To find the Moon's age on any day of the month in a given year, look for the day of the mean change in that year and month,

The Moon's distance gone from the Sun on all the different days of her age, from Change to Change.

Days.	Signs.	Deg.	Min.
1	0	12	11
2	0	24	23
3	1	6	34
4	1	18	46
5	2	0	57
6	2	13	9
7	2	25	20
8	3	7	32
9	3	19	43
10	4	1	54
11	4	14	6
12	4	26	17
13	5	8	29
14	5	20	40
15	6	2	52
16	6	15	3
17	6	27	15
18	7	9	26
19	7	21	37
20	8	3	49
21	8	16	0
22	8	28	12
23	9	10	23
24	9	22	35
25	10	4	47
26	10	16	58
27	10	20	9
28	11	11	20
29	11	23	32
½	0	0	0

month, in the Table, page 30 and 31, and then you have the days of the Moon's age by counting the number of days from the preceding change to the given day. Thus, ſuppoſe I would know the Moon's age on the 20th of May, in the year 1766; I look downward from May at the top of the Table, till I come right againſt 1766 in the left hand column, and againſt that year, under May, I find that the change was on the 9th of May; and as there are 11 between 9 and 20 (taking in the 20) it appears that the 20th of May was the 11th day of the Moon's age, in the year 1766.

Now, we are to find the time of the Moon's Southing (or coming to the Meridian) on the 11th day of her age: and the common rule for this is, to multiply the Moon's age by 4, and divide the product by 5; the quotient will be the hours (reckoned forward from noon) and the remainder, when there is any, multiplied by 12, will be the minutes of the time when the Moon

is

is on the Meridian. Thus, for the 11th day of the Moon's age, 4 times 11 is 44, which divided by 5 quotes 8 hours, and the remaining 4 being multiplied by 12 gives 48 minutes. So that, on the 11th day of the Moon's age, the Moon comes to the Meridian at 48 minutes paſt VIII in the evening.

We muſt now find the Moon's place in the Ecliptic on the abovementioned 20th of May, when ſhe is 11 days old.

By the Table, page 53, I find that on the 11th day of the Moon's age, ſhe is 4 ſigns 14 deg. 6 min. gone from the Sun. And by the Table on page 41, I find the Sun's place in the Ecliptic, on the 20th of May, to be ♉ 29° 14′; or in 29 degrees 14 minutes of Taurus, to which add the Moon's diſtance from the Sun, 4 ſigns, 14 degrees, 6 minutes, and the reckoning carries it to ♎ 13° 20′, or 13 deg. 20 min. of Libra; which is the Moon's place in the Ecliptic on the

the 20th of May, according to her mean motion.

Having thus found the Moon's place in the Ecliptic, we muſt next find her declination, which is the ſame as the Sun's, when in that place of the Ecliptic, namely 5 deg. 10 min. South, when in the 13 degree of Libra (ſee the Table page 43); ſo the Moon's Declination, on the 20th of May, when ſhe is 11 days old, is 5° 10′ South.

Now, to find her Semi-diurnal Arc on that day, ſuppoſe in the Latitude of 52 degrees, I look in the Table of Semi-diurnal Arcs (page 48) againſt 5 degrees of South Declination and under 52° of Latitude; where, in the Moon's column, I find her Semi-diurnal Arc to be 5 hours 49 minutes.

Laſtly, to the above-found time of the Moon's coming to the Meridian, 8 hours 48 minutes paſt Noon, add her Semi-diurnal Arc, 5 hours, 49 minutes, and the ſum will be the time of the Moon's ſetting; namely, at 14 hours 37 minutes paſt the Noon of the

the 20th of May; which is the 21ſt of May at 37 minutes after 2 o'clock in the morning. And her Semi-diurnal Arc, 5 hours 49 minutes, being ſubtracted from 8 h. 48 minutes, the time of her Southing, gives 2 hours 59 min. paſt Noon for the time of her riſing.

When the Moon's Semi-diurnal Arc is greater than the time of her coming to the Meridian, add 12 hours to that time, and then make the ſubtraction, and the remainder will give the time of the Moon's riſing; which will be in the morning.

In this proceſs, which may be gone through in two minutes, we have conſidered the Moon as moving always in the Ecliptic. But ſhe is ſometimes 5 degrees on the North ſide of it, and at other times as far on the South, which will affect the time of her riſing and ſetting about half an hour, on the parallel of London: more as the Latitude is farther North, and leſs

as it is ſarther South: but this difference can happen only twice in a lunation.

A Table ſhewing how much time is contained in any given number of mean Lunations. The Lunation being 29 *days,* 12 *hours,* 44 *minutes,* 3 *ſeconds,* 2‴, 58^{iv}, *or* 29.53059085108 *days.*

Lun.	Days. Decimals of a day.
1	29.53059085108
2	59.06118170216
3	88.59177255324
4	118.12236340432
5	147.65295425540
6	177.18354510648
7	206.71413595756
8	236.24472680864
9	265.77531765972

Although the Table ſeems to go no further than nine mean Lunations, yet it will do for any number from 1 Lunation to 900000000000, by removing the decimal point one place forward for tens of Lunations, two places forward for hundreds of Lunations, 3 places for thouſands, four places for tens of thouſands; and ſo on, as in the following Examples. For, if we wanted to know how much time is contained in 10 Lunations, then ſuppoſe

ſuppoſe a cypher put to 1 in the firſt column, to make it 10, and the decimal point in the firſt line to be put one place forward, it will be 295.3059085108, for the number of days and decimal parts of a day in 10 Lunations. The decimal parts may be reduced to the known parts of an integral day, by the common method of reducing decimals.

EXAMPLE I.

In 10 Lunations, Qu. *How much time?*

Lun.	Days. Decimals of a day.
10	295.3059085108
	mult. by 24$^{h.}$
	12236340432
	6118170216
	Hou. 7.3418042592
	mult. by 60$^{m.}$
	Min. 20.5082575520
	mult. by 60$^{s.}$
	Sec. 30.4954531200
	mult. by 60$^{th.}$
	Th. 29.4271872000

	d.	h.	m.	s.	th.
Anſwer	295	7	20	30	29.8

EXAMPLE II.

In 74212 *mean Lunations,* Qu. *How many days, hours, minutes, &c?*

Lun.	Days. Decimals of a day.
70000	2067141.3595756
4000	118122.36340432
200	5906.118170216
10	295.3059085108
2	59.06118170216
74212 Lun.	2191524.208240348 96 Days.

mult. by 24h.

83296139584
41648069792

Hours 4.997768375 04
mult. by 60m.

Min. 59.8661025 0240
mult. by 60s.

Sec. 51.96615014400
mult. by 60th.

thirds 57.96900864000

	y.	d.	h.	m.	s.	th.
Anſwer	6000	24	4	59	51	57. 969.

By reduction, 2191524 days contain 6000 Julian years and 24 days.

EXAMPLE III.

In 100000000000 *mean Lunations,* Qu. *How much time.*

Lun.	Days.	
100000000000	2953059085108	Anſwer.

In

In Example III, the number of cyphers annexed to 1 are equal to the number of decimal parts in the first line of the Table; and therefore the whole of that line becomes a whole number of integral days, without any fraction. So that, in 100,000,000,000 mean Lunations, there are just 2953059085108 days.

It is somewhat remarkable, that every 49th mean New Moon falls but 1 min. 30 sec. 34 thirds short of the same time of the day as before.

Example IV.

Lun.	Days. Decimals of a day.
40	1181.2236340432
9	265.77531765972
49	1446.99895170292
Lun.	Days. mult. by $24^{h.}$

399580681168
199790340584

Hou. 23.97484087008
mult. by $60^{m.}$

Min. 58.49045220480
mult. by $60^{s.}$

Sec. 29.42713228800
mult. by $60^{th.}$

Thirds. 25.6279372800

Which wants only 1 minute, 30 sec. 34.4th. of 1447 days.

A Table shewing how many mean Lunations are contained in any given quantity of time.

Years.	Lun. Decimals of a Lunation.	Hours.	Lun. Decimals of a Lunation.	Sec.	Lun. Decimals of a Lunation.
1	12.368530038627	1	0.0014109662	1	0.0000003919
2	24.737060077255	2	0.0028219345	2	0.0000007838
3	37.105590115882	3	0.0042328987	3	0.0000011758
4	49.474120154510	4	0.0056438649	4	0.0000015677
5	61.842650193137	5	0.0070548312	5	0.0000019597
6	74.211180231765	6	0.0084657974	6	0.0000023516
7	86.579710270392	7	0.0098767637	7	0.0000027435
8	98.948240309020	8	0.0112877299	8	0.0000031355
9	111.316770347647	9	0.0126986962	9	0.0000035274

Days.	Lun. Decimals of a Lunation.	Min.	Lun. Decimals of a Lunation.	Th.	Lun. Decimals of a Lunation.
1	00.033863189760	1	0.0000235161	1	0.0000000065
2	00.067726379520	2	0.0000470322	2	0.0000000131
3	00.101589659280	3	0.0000705483	3	0.0000000196
4	00.135452759040	4	0.0000940644	4	0.0000000262
5	00.169315948800	5	0.0001175805	5	0.0000000327
6	00.203179138560	6	0.0001410966	6	0.0000000392
7	00.237042328320	7	0.0001646127	7	0.0000000457
8	00.270905518080	8	0.0001881288	8	0.0000000522
9	00.304768707840	9	0.0002116449	9	0.0000000587

For tens of Julian years, days, hours, &c. remove the decimal points one place forward; for hundreds, two places; for thousands, three places; for tens of thousands, four places; and so on, as in the following Example. It appears by the first line of the above Table,

Table, that in 100000 Julian years (which contain 36525000 days) there are 1236853 mean Lunations, and .0038627, or $\frac{38627}{10000000}$ parts of a Lunation, which ſmall fraction may be neglected.

[*In common working, 'tis ſufficient to take in only four or five of the decimal figures.*]

Example. V.

In 6000 *Julian years,* 24 *days,* 4 *hours,* 59 *minutes,* 52 *ſeconds,* Qu. *How many mean Lunations?*

		Lun. Decimals.
Years	6000	74211.180231765
Days	20	0.677263795
	4	0.135452759
Hours	4	0.005643864
Min.	50	0.001175805
	9	0.000211645
Sec.	50	0.000019597
	2	0.000000784
Anſwer,		74212.000000014

More Examples would be ſuperfluous.

To explain the Phenomena of the Harveſt-Moon, by means of a common globe.

Make chalk-marks all round the globe on the Ecliptic, at $12\frac{1}{6}$ degrees from

from each other (beginning at Capricorn) which is equal to the Moon's mean motion from the Sun from day to day, near enough for your purpofe. Then elevate the North pole of the globe to the latitude of any place in Europe; fuppofe London, of which the latitude is 51½ degrees North.

This done, turn the ball of the globe round weftward, in the frame thereof; and you will fee that different parts of the Ecliptic make very different angles with the horizon, as thefe parts rife in the Eaft: and therefore, that in equal times, unequal portions of the Ecliptic will rife. About Pifces and Aries feven of the marks will rife in about two hours and an half, meafured by the motion of the index on the horary circle; but about the oppofite figns, Leo and Virgo, the index will go over eight hours in the time that 7 marks will rife. The intermediate figns will, more or lefs, partake of thefe differences, as they are more or lefs remote from them.

Hence

Hence it is plain, that when the Moon is in Pifces and Aries, the difference of her rifing will be no more than two hours and an half in feven days; but in Virgo and Libra it will be eight hours in feven days: and this happens in every lunation.

The Moon is always oppofite to the Sun when fhe is full; and the Sun is never in Virgo and Libra but in our Harveft-months, and therefore the Moon is never full in Pifces and Aries but in thefe months. And confequently, when the Moon is about her full in harveft, fhe rifes with lefs difference of time, for a week, than when fhe is full in any other month of the year.

Here we confider the Moon as moving always in the Ecliptic. But as fhe moves in an orbit which is inclined to the Ecliptic, her rifing when about the full in Harveft will fometimes not differ above an hour and 40 minutes through the whole of 7 days; and, at other times, it will differ three hours and an half, in a week, according to

the different poſitions of the Nodes of her orbit in the Ecliptic, in different years.

In our Winter, the Moon is in Piſces and Aries, about the time of her firſt quarter; and riſes about noon: but her riſing is not then taken notice of, becauſe the Sun is above the Horizon.

In Spring, the Moon is in Piſces and Aries, about the time of her change; and then, as ſhe gives no light, her riſing cannot be perceived.

In Summer, the Moon is in Piſces and Aries about her third quarter; and then, as ſhe riſes not till about midnight, her riſing paſſes unobſerved; eſpecially as ſhe is ſo much on the decreaſe.

But in Harveſt, Piſces and Aries are oppoſite to the Sun; and therefore the Moon is full in them at that time, and riſes nearly after Sun-ſet for ſeveral evenings together; which makes her riſing very conſpicuous at that time of the year, as it is ſo beneficial to the farmers, in affording them an imme-

diate

diate ſupply of light after the going down of the Sun, when they are reaping the fruits of the earth.

Rules for ſolving Aſtronomical Problems by the Logarithmic Tables of Sines and Tangents.

1. *The Sun's Longitude, or diſtance from the neareſt Equinoctial point (*viz.* the beginning of Aries or Libra) being given; to find his Declination.*

As Radius is to the Sine of the Sun's diſtance from the Equinoctial point, ſo is the Sine of his greateſt declination (23° 29′) to the Sine of his declination ſought.

2. *The Sun's Declination being given, to find his diſtance from the neareſt Equinoctial point, and conſequently his place in the Ecliptic.*

As the Sine of the Sun's greateſt declination is to the Sine of his preſent

declination, ſo is Radius to his diſtance gone from, or in going toward, the Equinoctial point required.

3. *The Sun's diſtance from the neareſt Equinoctial point being given, to find his right Aſcenſion.*

As Radius is to the Co-ſine of the Sun's greateſt declination, ſo is the Tangent of his diſtance from Aries or Libra, to his right aſcenſion therefrom.

4. *The Latitude of the place, and the Sun's Declination being given, to find his Aſcenſional difference.*

As the Co-tangent of the latitude is to the Tangent of the Sun's declination, ſo is Radius to the Sine of his aſcenſional difference required.

5. *The Latitude of the place and the Sun's declination being given, to find his Amplitude, or the number of degrees he riſes and ſets from the Eaſt and Weſt.*

As the Co-ſine of the latitude is to the Sine of the Sun's declination, ſo is the Sine of his diſtance from Aries or Libra to the Sine of his Amplitude.

6. *The Sun's right Aſcenſion and his greateſt Declination being given, to find the Angle of the Ecliptic and Meridian.*

As Radius is to the Sine of the Sun's greateſt declination, ſo is the Co-ſine of his right aſcenſion to the Co-ſine of the angle ſought.

7. *The*

7. *The Latitude of the Place and the Sun's declination being given, to find the Sun's Altitude when he is due East or West.*

As the Sine of the latitude is to the Sine of the Sun's declination, ſo is Radius to the Sine of his altitude when due Eaſt or Weſt.

N. B. By this problem, a true meridian line may be drawn in Summer, when the Sun riſes before he comes to the Eaſt, and paſſes by the Weſt before he ſets. For, if a long upright wire be ſet in a truly level board, the ſhadow of the wire will run Weſtward on the board when the Sun is due Eaſt, and Eaſtward when the Sun is due Weſt; which will be at the inſtant when his altitude, obſerved by a quadrant, agrees with what the problem makes it. And then, if two points are marked in the line of the ſhadow, and a ſtraight

ſtraight line be drawn through them on the board, and this line be croſſed at right angles, in any point, by another ſtraight line, that line, will be a true meridian line; and if the wire be placed perpendicularly in the interſection of theſe two lines, the ſhadow of the wire will cover the meridian line when the Sun is on the meridian of the place. This may be done beſt of all about the Summer ſolſtice, becauſe the Sun changes his altitude faſteſt, and his declination ſloweſt, about that time.

8. *The Latitude of the place and the Sun's declination being given, to find the Sun's altitude at ſix o'clock in the Morning or Evening.*

As Radius is to the Sine of the Sun's declination, ſo is the Sine of the latitude to the Sine of the Sun's altitude at ſix o'clock.

By this problem, you may know when it is exactly ſix o'clock by the Sun,

Sun, and conſequently how to place a Sun-dial true at that inſtant, provided it be done in the Summer time, when the Sun is above the horizon at ſix. For, if you keep watching, and obſerving the Sun's altitude with a quadrant, when you judge the time to be a little before ſix, till you find his altitude agrees with what the problem makes it, you are ſure that it is then preciſely ſix o'clock by the Sun; to which time, ſet your watch, and then you may ſet it to the true equal time by a common Equation-table, which ſhews how much the Sun's time is faſter or ſlower than the equal time, every day of the year.

N. B. In all obſervations of the Sun's altitude, you muſt ſubtract the refraction of the Sun's rays by the Atmoſphere from the obſerved altitude; for otherwiſe you will not have it true. And for this purpoſe, I ſhall ſubjoin a Table of refractions at the end of theſe problems, to

ſhew

ſhew how much leſs the true altitude is, than the obſerved altitude; and when it is ſo much leſs than the problem gives, as is equal to the quantity of refraction at the time of the obſerved altitude, you have the altitude true.

And here, with regard to the placing of Sun-dials, I muſt make an obſervation, that may perhaps ſeem a very odd one to moſt people; which is, that if the Dial be made according to the ſtrict rules of calculation, and be truly ſet at the inſtant when the Suns is on the Meridian; it will be a minute too faſt in the Forenoon, and a minute too ſlow in the Afternoon, by the ſhadow of the Stile; for the edge of the ſhadow that ſhews the time is even with the Sun's foremoſt edge all the time before Noon, and even with his hindmoſt edge all the Afternoon, on the Dial. But it is the Sun's center that determines the time in the (ſuppoſed) Hour circles of the heaven. And as the Sun is half a degree in breadth, he takes two minutes

to move through a ſpace equal to his breadth; ſo that there will be two minutes at Noon in which the ſhadow will have no motion at all on the Dial. Conſequently, if the Dial be ſet true by the Sun in the Forenoon, it will be two minutes too ſlow in the Afternoon; and if it be ſet true in the Afternoon, it will be two minutes too faſt in the Forenoon.

The only way that I know of to remedy this, is to ſet every hour and minute diviſion on the Dial one minute nearer XII than the calculation makes it to be.

The Sun moves 15 degrees in one hour, 30 degrees in two, 45 in three, 60 in four, 75 in five hours, and 90 in ſix, with reſpect to the equator; but, in an oblique ſphere, the motion of the ſhadow, either on a horizontal or vertical plane, is very different. To find the degrees and minutes of a degree of the hour-diſtances from XII, on a horizontal Dial, ſay, as Radius is to the ſine of the latitude of the place (which

(which is the ſame as the angle of the ſtiles height) ſo is the Tangent of 15 degrees, and of 30, and 45, 60, 75, and 90, to the Tangent of the diſtance (in degrees and minutes) of XI and I, X and II, IX and III, VIII and IV, VII and V, VI and VI, from XII on the Dial.

The ſame calculation ſerves for erect South Dials, only uſing the Co-latitude inſtead of the latitude, for the hours and height of the ſtile.

9. *The Latitude of the Place and the Sun's Declination being given, to find the Sun's Azimuth from the North at ſix o'clock.*

As the Co-ſine of the latitude is to Radius, ſo is the Co-tangent of the Sun's declination to the Tangent of his Azimuth from the North at ſix.

10. *The Sun's Altitude, Declination, and time of the day, being given; to find the Snn's Azimuth from the North at that time.*

As the Co-ſine of the Sun's altitude is to the ſine of the time from Noon (converted into degrees), ſo is the Co-ſine of the Sun's declination to the ſine of his Azimuth from the North.

11. *The Latitude of the place and the Sun's Declination being given, to find the time of the Sun's riſing and ſetting.*

Find the aſcenſional difference by Problem 4. Then, the degrees of the aſcenſional difference being converted into time, ſubtract that time from 6 hours, when the Sun is in ♈, ♉, ♊, ♋, ♌, and ♍, and the remainder will be the time of Sun-riſing, and added to 6 will be the time of Sun-ſetting. But, when the

the Sun is in ♎, ♏, ♐, ♑, ♒, and ♓, the aſcenſional difference added to 6 hours gives the time of Sun-riſing, and ſubtracted therefrom gives the time of his ſetting. How to find his Amplitude at riſing and ſetting, is already ſhewn by the 5th Problem.

12. *The Latitude of the Place and the Sun's Declination being given, to find the Sun's Meridian Altitude.*

Subtract the latitude from 90 degrees, and the remainder will be the Co-latitude. Then, if the latitude and declination be both North or both South, the ſum of the declination and Co-latitude is the Sun's Meridian altitude. But when either of theſe is North and the other South, their difference is the Meridian altitude.

To find the Sun's Altitude at any time of the day, by the Shadow of an upright object on a horizontal Plane.

As the length of the ſhadow is to the height of the object, ſo is Radius to the Tangent of the Sun's altitude at the time of obſervation.

The Latitude of the place, the Sun's Meridian Altitude, and preſent Altitude, being given; to find the time of the day.

As Radius is to the Co-ſine of the Sun's declination, ſo is the Co-ſine of the latitude to a fourth ſine: and, as that fourth ſine is to Radius, ſo is the difference between the Sun's meridian altitude and his preſent altitude to the verſed ſine of the time from Noon.

The intended Aſtronomical Problems being finiſhed, we now give the promiſed Table of Refractions. *See pag.* 72.

A Table ſhewing the Refractions of the Sun, Moon, and Stars, adapted to their Apparent Altitudes.

Appar. Altit.		Refraction.		Ap Alt.	Refraction.		Ap. Alt.	Refraction.	
°	′	′	″	°	′	″	°	′	″
0	0	33	45	21	2	18	56	0	36
0	15	30	24	22	2	11	57	0	35
0	30	27	35	23	2	5	58	0	34
0	45	25	11	24	1	59	59	0	32
1	0	23	7	25	1	54	60	0	31
1	15	21	20	26	1	49	61	0	30
1	30	19	46	27	1	44	62	0	28
1	45	18	22	28	1	40	63	0	27
2	0	17	8	29	1	36	64	0	26
2	30	15	2	30	1	32	65	0	25
3	0	13	20	31	1	28	66	0	24
3	30	11	57	32	1	25	67	0	23
4	0	10	48	33	1	22	68	0	22
4	30	9	50	34	1	19	69	0	21
5	0	9	2	35	1	16	70	0	20
5	30	8	21	36	1	13	71	0	19
6	0	7	45	37	1	11	72	0	18
6	30	7	14	38	1	8	73	0	17
7	0	6	47	39	1	6	74	0	16
7	30	6	22	40	1	4	75	0	15
8	0	6	0	41	1	2	76	0	14
8	30	5	40	42	1	0	77	0	13
9	0	5	22	43	0	58	78	0	12
9	30	5	6	44	0	56	79	0	11
10	0	4	52	45	0	54	80	0	10
11	0	4	27	46	0	52	81	0	9
12	0	4	5	47	0	50	82	0	8
13	0	3	47	48	0	48	83	0	7
14	0	3	31	49	0	47	84	0	6
15	0	3	17	50	0	45	85	0	5
16	0	3	4	51	0	44	86	0	4
17	0	2	53	52	0	42	87	0	3
18	0	2	43	53	0	40	88	6	2
19	0	2	34	54	0	39	89	0	1
20	0	2	26	55	0	38	90	0	0

The

The Description of an Instrument for solving many Astronomical Problems; finding the Hour-distances from XII *on horizontal and vertical Dials; forming spherical Triangles, and solving the Cases depending thereon,* &c.

Mr. Mungo Murray, Shipwright, contrived a very useful instrument several years ago, which he calls *The Armillary Trigonometer:* and I had it some months by me in the year 1757. Since that time, he shewed me a pasteboard model of an instrument, much of the same sort, but of a much smaller size; which, I believe, he has not yet made, either of wood or metal. And, as it is a thing that deserves well to be known, on account of its great utility, I have made it of wood, as represented in Plate I. The only addition that I have made to Mr. Murray's scheme, is a circular scale of the Sun's declination for the different days of the year, to save the trouble of referring

to

to Tables of the Sun's declination in printed books; as it is one of the *data* that muſt be had in ſolving moſt of the following Problems, which are only a few of thoſe that may be ſolved by it.

The upright circular board *A* is 12 inches in diameter, and one inch in thickneſs. It ſtands on the horizontal foot *B*.

On the left hand ſide of this board is a flat ſemicircle *C*; which is made of box wood, and is pinned faſt to the board *A*.

To this ſemicircle is joined ſuch another, *D*, by two hinges, at the zenith and nadir (ſo marked in the figure); and is moveable on theſe hinges. When *D* is put down flat to the board, it and the other make a flat circular ring; on which, the months and days of the year are laid down, and all the degrees of the Sun's North and South declinations anſwering thereto: within which, the four quadrants of the circle are divided into 90 degrees each.

In the upright board *A*, the ends of the horizontal ſemicircle *E* are fixed. This ſemicircle ſtands at right angles to the plane of the board, parallel to the foot *B*; and is divided forwards and backwards into 32 equal parts, for the points of the compaſs; within which the two quadrants are divided into 90 degrees each, numbered from the South and North points to the Eaſt and Weſt, at *E. W.* where the numbers end at 90.

Within the two ſemicircles *C* and *D* (when *D* is put down) is the flat board *F*, whoſe ſurface lies even with the ſurfaces of theſe two ſemicircles; and which is moveable, round the fixed pin *f* in the center. On this board is a diameter line (marked *Axis*) which repreſents the axis of the world, and terminates in the North and South poles; where two hinges join the moveable ſemicircle *G* to the board. This ſemicircle is divided, upwards and downwards, from the middle to the North and South poles, into twice 90 degrees, for

for all the North and South declinations of the Sun, Moon, and Stars.

In the moveable board *F*, the ends of a ſemicircle *H* are fixed. The plane of this ſemicircle is at right angles to the plane of the board, and alſo to the plane of the ſemicircle *G* in all poſitions. It is firſt divided into twelve equal parts, where the hours are doubly laid down: and then, each hour is ſubdivided to every fifth minute. The outermoſt hours are thoſe from midnight to Noon, and the innermoſt are the hours from Noon to Midnight.

A quadrant *I*, whoſe ſurface is even with the ſurface of the great board *A*, is divided into 90 degrees, numbered upward from the horizon to the zenith.

As theſe ſemicircles anſwer all the purpoſes of whole circles, in the inſtrument; we ſhall call *D* the *vertical circle*, *E* the *horizon*, *G* the *hour-circle*, and *H* the *equator*. There is a notch in *D*, which receives *E*; and the innermoſt edge of *D* goes cloſe to the outermoſt edge *G*, whoſe innermoſt

edge touches the outermoſt edge of *H*, in all poſitions.

The latitude of the place, and the day of the month being given; to rectify the Inſtrument for uſe.

In the following Problems, we ſhall always ſuppoſe the latitude of the given place to be North. Therefore, turn the moveable board *F* till the North pole comes to the latitude of the place, on the quadrant *I*; then, find the Sun's declination for the given day of the year, on the ſemicircles *C* or *D*; and, as that declination is North or South, mark it with chalk, North or South of the equator *H*, on the moveable hour-circle *G*; and the inſtrument will be rectified: and remember that it muſt always be ſo, in each of the following Problems, except the 9th, and 11th.

Prob. I.

To find the time of the day, either in the Forenoon or Afternoon; and the Sun's true Azimuth from the South at that time.

Obſerve the Sun's altitude with a quadrant. Then, move the vertical circle *D*, and the hour-circle *G*, till the Sun's obſerved altitude (above the horizon *E*) on the former, coincides with his declination on the latter: and then, the circle *G* will cut the time of the day in *H* when the obſervation was made; and the number of degrees reckoned from the South point of the horizon *E* to the vertical circle *D*, will be the Sun's true Azimuth from the South at that time.

Prob. II.

To find the variation of the compaſs.

The Sun's true Azimuth being found by the foregoing Problem, compare it with

with the azimuth ſhewn by the compaſs at the time of obſervation: and the difference will be the variation of the compaſs at the place where the obſervation was made.

PROB. III.

The time of the day being given; to find the Sun's altitude and azimuth at that time.

Put the hour-circle *G* to the given time on *H*; and, keeping it there, move the vertical circle *D* till it cuts the Sun's declination in *G*; and the interſection will cut the Sun's altitude above the horizon in *D*, and *D* will cut the Sun's azimuth from the South in *E*.

PROB. IV.

To find the time of the Sun's riſing and ſetting, on any day of the year, in any given North latitude leſs than 66½ *degrees.*

The reaſon for confining this Problem within 66½ degrees is, that in greater

greater latitudes, the Sun continues ſeveral natural days (of 24 hours each) above the horizon in Summer, without ſetting; and the time is the longer as the place is the nearer to the pole. At the poles of the earth, the Sun is continually above the horizon for the Summer half year, and continually below it for the Winter half. To ſolve this limited Problem, turn the hour-circle *G* till the Sun's declination thereon, for the given day, comes to the horizon *E*; and then, *G* will cut *H* in the time of the Sun's riſing, among the outermoſt hours; and the time of his ſetting, among the innermoſt.

PROB. V.

To find when the Morning Twilight begins, and when the Evening Twilight ends.

When the Sun is juſt 18 degrees below the horizon in the Morning, the

Twilight

Twilight begins; and when he is 18 degrees below the horizon in the Evening, the Twilight ends. Therefore, mark the 18 degree below the horizon *E* in the vertical circle *D*; and mark the Sun's declination for the given day in the moveable hour-circle *G*. This done, turn *D* and *G* on their hinges till you find the 18th degree below the horizon on *D* cuts the declination on *G*: and then *G* will cut *H* in the time when the Morning Twilight begins, among the outermoſt hours; and the time when the Evening Twilight ends, among the innermoſt.

N. B. When the point of the Sun's declination, in the Summer months, does not go ſo far as 18 degrees below the horizon, at Midnight; the Twilight continues all the Night.

PROB. VI.

Prob. VI.

A place being given in the North frigid zone (that is, in more than 66½ degrees of North latitude) to find on what day of the year the Sun begins to ſhine conſtantly on that place without ſetting; and how long he continues to do ſo.

The pole being elevated to the latitude of the place, put down the moveable hour-circle *G* quite flat to the board *A*; and then, obſerve what degree, or point, of North declination on *G* cuts the horizon *E*. When the Sun is at that point of declination, before the 21ſt of June, he begins to go on, without ſetting; and continues to do ſo till he comes to the like point of declination after the 21ſt of June. Therefore, the two days, before and after the 21ſt of June, which anſwer to the ſaid point of declination in the ſcale of months, give the ſolution of the Problem: *that* before the 21ſt of June

being the day on which the Sun begins to go round with ſetting; and he continues to do ſo, till the *other* after the 21ſt of June, on which he begins to ſet; and then to riſe and ſet as at other places.

PROB. VII.

To find how long the Twilight continues at the poles of the Earth.

The continuance is equal at each pole, but at contrary times of the year: ſuppoſe therefore we take it for the North pole.

At the North pole, while the Sun is above the horizon, his altitude is equal to his declination North: and while he is below the horizon, his depreſſion is equal to his declination South; and his South declination begins on the 23d of September, and ends on the 20th of March. But, as his South declination is within 18 degrees from the 23d of September to the 13th

of November, the Twilight continues all that time after the Sun ſets. And as it is within 18 degrees from the 29th of January to the 20th of March, there is Twilight all that time at the North pole before the Sun riſes to it.

PROB. VIII.

To find the Sun's depreſſion below the horizon, at any time of the night, in any given latitude leſs than $66\frac{1}{2}$ *degrees.*

The inſtrument being rectified, bring the moveable circle *G* to the given time of the night in *H*; then, move the vertical circle *D* till it cuts the Sun's declination for the given day in *G*, and the declination in *G* will cut the number of degrees of the Sun's depreſſion below the horizon at that time, in *D*.

Prob. IX.

To find in what North latitude the longeſt day is of any given length leſs than 24 hours.

In Northern latitudes, the longeſt day is when the Sun's declination is 23½ degrees North. Divide the given length by 2, and the quotient will give the time of Sun-ſetting: to which time, place the circle *G* among the innermoſt hours on *H*; and then, turn the moveable board *F* till 23½ degrees of North declination on *G* comes down to the horizon *E*; and the elevation of the pole above the horizon will ſhew the latitude of the place, in the quadrant *I*.

Prob. X.

To find the Sun's amplitude at riſing and ſetting, in any given latitude leſs than 66½ *degrees.*

The time of the Sun's riſing and ſetting being found by Prob. IV. count the number

number of degrees on the horizon *E*, at which he riſes and ſets from the Eaſt and Weſt points; and *that* number will be the Sun's amplitude.

Prob. XI.

The length of the longeſt day being given, at any place whoſe latitude is North; to find the latitude of that place.

If the given length be leſs than 24 hours, ſubtract its half from 12 hours; and the remainder will be the time of Sun-ſetting on that day. To which time, place the moveable hour-circle *G*, among the innermoſt hours on *H*; and then, turn the moveable board *F* till $23\frac{1}{2}$ degrees of North declination on *G* comes down to the horizon *E*, and the pole will then point out the latitude of the place, in the quadrant *I*: the elevation of the pole above the horizon of any place being always equal to the latitude of the place.

If the length of the longeſt day conſiſts of ſeveral natural days, of 24 hours each; take the Sun's altitude by a quadrant when he is due North, on the 21ſt of June; at which time his declination is $23\frac{1}{2}$ degrees North. Then, put down the ſemicircles *D* and *G*, flat to the board *A*; and turn the moveable board *F* till $23\frac{1}{2}$ degrees of North declination on *G* cuts the Sun's obſerved altitude on *D*: and then, the North pole will point to the latitude of the place, in the quadrant *I*.

PROB. XII.

In the Summer months, to find an Eaſt and Weſt line; and conſequently a Meridian line, for a place of any given latitude.

This is beſt done about the time of the Summer ſolſtice; becauſe the Sun's declination changes ſlower about that time than any other in Summer: and it cannot be done in the Winter-half

of

of the year, by the inſtrument; becauſe, during that time, the Sun is always paſt the Eaſt before he riſes; and he ſets before he comes to the Weſt.

Having a wire ſet upright in a level board, on which the Sun may ſhine when he is due Eaſt or due Weſt, as already mentioned (pag. 70) and the inſtrument being rectified, bring the vertical circle *D* to the Eaſt and Weſt point of the horizon *E*, and turn the moveable hour-circle *G* till the Sun's declination thereon, for the given day, comes to the vertical circle *D*; and the declination in *G* will cut the Sun's altitude in *D* when he is due Eaſt or due Weſt on that day. For the reſt of the operation, ſee pag. 70 and 71.

Prob. XIII.

To find the diſtances of all the Forenoon and Afternoon hours from XII, *on a horizontal dial for any given Latitude.*

Elevate the pole, in the quadrant *I*, to the latitude of the given place; and

bring

bring the moveable hour-circle *G* successively to all the outermoſt hours on the equator *H*; and *G* will cut the diſtances of all the Foreoon hours from XII on the horizon *E*, as you bring it to the like hours on the equator. The Afternoon hours being at the ſame diſtances from XII as the Forenoon hours are, by having the latter we have alſo the former.

N. B. When $23\frac{1}{2}$ degrees of North declination on *G* comes to the horizon *E*, *G* will cut *H* in the time of the Sun's riſing and ſetting on the longeſt day; and conſequently will limit the number of hours to be put upon the dial. By the ſame method, the half hour and quarter diſtances from XII may be found, for all the hours on the dial.

PROB. XIV.

Prob. XIV.

To find the diſtances of the Forenoon and Afternoon hours from XII, *on a vertical South dial for any given latitude.*

The pole being elevated in the quadrant *I*, to the latitude of the given place, bring the vertical circle *D* to the Eaſt and Weſt point of the horizon *E* (at E. W.) This done, bring the moveable circle *G* ſucceſſively to all the outermoſt hours on *H*, from XII to VI; and *G* will cut *D* in all theſe hour-diſtances from XII, reckoned downward from the zenith to the horizon. Theſe are the Forenoon hours: and as all the Afternoon hour diſtances from XII to VI are the ſame as the Forenoon hour-diſtances, 'tis needleſs to work for them by the inſtrument.

N. B. On all erect direct South dials, the Forenoon hours begin at VI in the Morning; and the Afternoon

hours end at VI in the Evening: for the Sun never ſhines more than twelve hours on any dial whoſe plane is perpendicular to the horizon.

The meridian, or twelve o'clock line, on theſe two dials, muſt be made as broad as the ſtile is thick. The angle of the ſtile's height muſt be equal to the latitude of the place for which the horizontal dial is made; and the angle of the ſtile's height in an erect direct South dial muſt be equal to the Co-latitude of the place.

Having thus found the hour-diſtances from XII by the inſtrument, ſet them off with your compaſſes, by a line of chords, from the twelve o'clock line on your dial plates; which lines being made as broad as the ſtile is thick, ſet off the Forenoon hours from the edge of the twelve o'clock line, which is to the Forenoon ſide of the dial; and the Afternoon hours from the Afternoon edge of that line.

The

The ſix o'clock line is perpendicular to the meridian line on theſe dials. It muſt be drawn before you begin to ſet off the hour-diſtances on the dial: and the centers of the two quadrantal arcs (taken equal to the chord of 60 degrees on your ſcale) muſt be in thoſe points of the edges of the broad meridian line, where the ſix o'clock line interſects it. And the broad edge of the ſtile, that ſhews the time by the ſhadow, muſt riſe from thoſe points in the dial which were made the centers of the above quadrantal arcs on which the hour-diſtances are ſet off from XII.

PROB. XV.

To find the diſtances of the Forenoon and Afternoon hours from XII, *on a vertical dial, declining from the South toward the Eaſt or Weſt, by any given number of degrees.*

Let us ſuppoſe that face of the dial muſt decline (or turn away) 30 degrees

from the South toward the Eaſt; then, 'tis evident, that the Afternoon edge of the plane of the dial is 30 degrees from the Eaſt toward the North, and the Forenoon edge thereof is 30 degrees from the Weſt toward the South.

Therefore, count the 30 degrees of declination from the Eaſt point of the horizon *E* (at 90, under E. W.) toward the North point; and where the reckoning ends (*viz.* at 60 degrees from the North), place the vertical circle *D* in the horizon; and *D* will repreſent the plane of the declining dial. Then, to find the diſtances of the Forenoon hours from XII on the dial, bring the moveable hour-circle *G* ſucceſſively to all the Forenoon hours (which are the outermoſt) on the equator *H*, as XI, X, IX, *&c.* till *G* comes to the horizon *E* in the point where the vertical circle *D* interſects it. And in doing this, *G* will cut the diſtances on *D* of all the Forenoon hours from XII, that muſt be put upon the dial; theſe diſtances

being

being reckoned downward on *D*, from the zenith to the horizon.

Then, becauſe we have only ſemicircles in the inſtrument, to find all the hour diſtances by; and as the Afternoon hours are not equi-diſtant with the Forenoon hours from XII on declining dials; let the Afternoon hours I, II, III, &c. be reckoned among the outermoſt on the equator *H* from the right hand toward the left, and let their diſtances from XII be taken upward on the vertical circle *D*, from the nadir toward the horizon *E*. To find theſe diſtances, bring the moveable hour-circle *G* ſucceſſively to the hours I, II, III, &c. (which are outermoſt) on the equator, till it comes to *that* point of the horizon *E* where the vertical circle *D* interſects it; and *G* will cut *D* in the diſtances (reckoned upward from the nadir) of all the Afternoon hours that muſt be inſerted on the dial.

Having thus found the diſtances from XII, of all the Forenoon and Afternoon hours that muſt be inſerted

on

on the declining dial, and wrote them down; draw a ſingle line (no thicker than the other hour lines) acroſs the plane of the dial, perpendicular to that edge of it which muſt be the lowermoſt, and be parallel to the horizon, when the dial is ſet; and that line, perpendicular to the horizon, will be the Meridian or twelve o'clock line of the dial.

Near the uppermoſt end of that line, aſſume a point for the center of the dial; and, having taken 60 degrees from the line of chords in your compaſſes, ſet one foot in the center-point, and with the other foot deſcribe a ſemicircle on the dial-plane; and thereon ſet off all the above-found diſtances from the XII o'clock line; and place the hours at theſe diſtances accordingly.

Then, to find the diſtance of the ſubſtile (or line on which the ſtile muſt ſtand) from the meridian line of the dial; bring the moveable hour circle *G* to as many degrees from the Eaſt toward the South point of the horizon *E* as the vertical circle *D* ſtands at

from

from the Eaſt toward the North point; and the circles *D* and *G* will croſs each other at right angles. Then, the number of degrees on *D*, which are intercepted between *G* and the zenith, will be the angle that the ſubſtile makes with the meridian of the dial; which muſt be ſet off from XII, among the Forenoon hours, becauſe the face of the dial declines from the South toward the Eaſt. And the number of degrees on *G*, which are intercepted between the vertical circle *D* and the North pole, will be the angle of the ſtile's height.

The ſubſtile line muſt be drawn to the center point of the dial, that is, to the center of the ſemicircle on which the hour-diſtances were ſet off from XII; and the edge of the ſtile that ſhews the hours by the ſhadow, muſt begin to riſe from the dial at the center point.

In this dial, the ſtile muſt be very thin, or elſe have a ſharp edge. If the dial declines Weſtward from the South, the vertical circle *D* muſt be placed

placed as many degrees from the Eaſt point of the horizon, toward the South, as the dial declines: and then, the hour-diſtances from XII are to be found in the ſame manner as above deſcribed.

In Eaſt-declining dials, the ſubſtile falls among the Forenoon hours; and in Weſt-declining dials, among the Afternoon hours. For, in all kinds of dials, when they are rightly ſet, the edge of the ſtile, that caſts the ſhadow for ſhewing the time of the day, muſt be parallel to the earth's axis.

Every one who reads this, and underſtands the uſe of the globes, will eaſily ſee that theſe are only a few of the Problems that may be ſolved by this inſtrument. And a bare view of the figure of it is ſufficient to ſhew, that any ſpherical triangle may be readily formed and ſolved by it; and conſequently, all the Problems that depend on ſpherical trigonometry. That juſtice might have been done to it, I wiſh Mr. Murray himſelf had deſcribed it, and ſhewn all its uſes.

To

To know, by the Stars, whether a clock goes true or not.

A Table ſhewing the daily accelerations of the Stars.

Days.	Accelerations. Hou.	Min.	Sec.	Th.
1	0	3	55	54
2	0	7	51	48
3	0	11	47	42
4	0	15	43	36
5	0	19	39	30
6	0	23	35	24
7	0	27	31	18
8	0	31	27	12
9	0	35	23	6
10	0	39	19	0
11	0	43	14	54
12	0	47	10	48
13	0	51	6	42
14	0	55	2	36
15	0	58	58	30
16	1	2	54	24
17	1	6	50	18
18	1	10	46	12
19	1	14	42	6
20	1	18	38	0
21	1	22	33	54
22	1	26	29	48
23	1	30	25	42
24	1	34	21	36
25	1	38	17	30
26	1	42	13	24
27	1	46	9	18
28	1	50	5	12
29	1	54	1	6
30	1	57	57	0

The ſtars make 366 revolutions from the Meridian to the Meridian again, or from any point of the compaſs to the ſame point again, in 365 days; and thererefore they gain a 365th part of a revolution every 24 hours of mean ſolar time.

Conſequently, if you mark the preciſe moment ſhewn by a clock, when any ſtar vaniſhes behind a chimney, or other object, as ſeen through a ſmall hole in a thin plate of metal, fixed in a window-ſhutter; and do this for ſeveral nights together (as ſuppoſe 20) if at the end of that time the ſtar vaniſhes

as much ſooner than it did the firſt night, by the clock, as anſwers to the accelerations for ſo many days in the Table; the clock goes true; otherwiſe not. If the difference between the clock and ſtar be leſs than the Table ſhews, the clock goes too faſt; if greater, it goes too ſlow; and muſt be regulated accordingly, by letting down or raiſing up the ball of the pendulum, by little and little, till you find it keep to true equal time.

Thus, ſuppoſe the ſtar ſhould diſappear behind the chimney any night when it is XII by the clock; and that, on the 20th night afterward, the ſame ſtar ſhould diſappear when it is 41 minutes, 22 ſeconds, paſt X by the clock, which ſubtracted from XII h. 0 m. 0 ſ. leaves remaining 1 hour, 18 minutes, 38 ſeconds, for the time the ſtar is then faſter than the clock; look in the Table, and againſt 20, in the left-hand column, you will find the acceleration of the ſtar to be 1 hour, 18 min. 38 ſeconds; agreeing exactly with what

what the difference between the clock and ſtar ought to be; which ſhews that the clock meaſures true equal time.

Dr. *Deſaguliers* informs us, that the length of a pendulum (from the point of ſuſpenſion to the center of oſcillation) that ſwings ſeconds in the latitude of London, is 29.128 inches. Now, to find the length of a pendulum that ſhall make any other given number of vibrations in the ſame latitude, in a minute; ſay, as the ſquare of the given number of vibrations is to the ſquare of 60, ſo is 39.128 inches (the length of the ſtandard) to the length in inches of the pendulum ſought.

By this rule, the following Table is calculated, for all numbers of vibrations in a minute, from 1 to 180, ſerving for the latitude of London. And, by the next Table that follows (page 110) the pendulum may be corrected for any other latitude.

A Table, shewing of what length a Pendulum must be, to make any given number of Vibrations in a minute, from 1 *to* 180, in Lat. 51° 30′.

Vibrations.	Length of the Pendulum. Feet.	Inches.	Vibrations.	Length of the Pend. Feet.	Inches.	Vibrations.	Length of the Pend. Feet.	Inches.
1	11738	4.800	31	12	2.577	61	3	1.852
2	2934	7.200	32	11	5.637	62	3	0.644
3	1304	3.200	33	10	9.358	63	2	11.490
4	733	7.800	34	10	1.852	64	2	10.387
5	469	6.432	35	9	6.998	65	2	9.340
6	326	0.800	36	9	0.681	66	2	8.337
7	239	6.710	37	8	6.894	67	2	7.381
8	183	4.800	38	8	1.543	68	2	6.465
9	144	11.022	39	7	8.611	69	2	5.586
10	116	8.608	40	7	4.038	70	2	4.747
11	97	0.139	41	6	11.736	71	3	3.943
12	80	11.255	42	6	7.753	72	2	3.172
13	69	2.538	43	6	4.074	73	2	2.433
14	59	5.566	44	6	0.759	74	2	1.723
15	52	2.048	45	5	9.561	75	2	1.042
16	45	10.234	46	5	6.569	76	2	0.387
17	40	7.408	47	5	3.767	77	1	11.758
18	36	2.755	48	5	1.137	78	1	11.152
19	32	6.196	49	4	10.607	79	1	10.571
20	29	4.154	50	4	8.344	80	1	10.009
21	26	7.639	51	4	6.156	81	1	9.469
22	24	3.034	52	4	4.093	82	1	8.949
23	22	2.277	53	4	2.146	83	1	8.447
24	20	4.550	54	4	0.169	84	1	7.963
25	18	8.377	55	3	10.465	85	1	7.496
26	17	4.374	56	3	8.818	86	1	7.045
27	16	1.238	57	3	7.355	87	1	6.610
28	14	11.669	58	3	5.873	88	1	6.190
29	13	11.492	59	3	4.466	89	1	5.771
30	13	0.512	60	3	3.128	90	1	5.390

The

The Pendulum Table concluded.

Vibrations.	Length of the Pend. Feet.	Inch.	Vibrations.	Length of the Pend. Inches.	Vibrations.	Length of the Pend. Inches.
91	1	5.040	121	9.620	151	6.175
92	1	4.642	122	9.464	152	6 096
93	1	4.252	123	9.310	153	6.017
94	1	3.939	124	9.161	154	5.939
95	1	3.597	125	9.015	155	5.863
96	1	3.284	126	8.872	156	5.788
97	1	2.971	127	8.727	157	5.715
98	1	2.670	128	8.597	158	5.643
99	1	2.372	129	8.464	159	5.572
100	1	2.068	130	8.335	160	5.502
101	1	1.800	131	8.208	161	5.434
102	1	1.539	132	8.084	162	5.367
103	1	1.277	133	7.963	163	5.301
104	1	1.006	134	7.844	164	5.238
105	1	0.776	135	7.728	165	5.211
106	1	0.536	136	7.615	166	5.112
107	1	0.303	137	7.510	167	5.051
108	1	0.076	138	7.396	168	4.991
109	0	11.856	139	7.291	169	4.932
110	0	11.641	140	7.187	170	4.874
111	0	11.433	141	7.085	171	4.817
112	0	11.211	142	6.985	172	4.761
113	0	11.031	143	6.888	173	4.707
114	0	10.838	144	6.793	174	4.652
115	0	10.561	145	6.699	175	4.599
116	0	10.368	146	6.608	176	4.547
117	0	10.281	147	6.518	177	4.496
118	0	10.117	148	6.431	178	4.445
119	0	9.954	149	6.345	179	4 396
120	0	9.782	150	6.260	180	4.347

Length for 300 Vibrations in a min. 1 inch and .565 parts of an inch.

To make the lengths of pendulums anſwer to the intended numbers of Vibrations, as in this Table; the weight of the ball, in all caſes, ſhould bear the ſame proportion to the weight of the pendulum rod, as the weight of the ſtandard ball bears to the weight of its rod.

A Table

A Table, ſhewing how much a Pendulum that ſwings Seconds at the Equator would gain every 24 hours in different Latitudes; and how much the Pendulum would need to be lengthened in theſe Latitudes, in order to make it ſwing Seconds therein.

Latitude of the Place.	Time gained in one Day.	Lengthening of the Pendulum to ſwing Seconds.		Latitude of the Place.	Time gained in one Day.	Lengthening of the Pendulum to ſwing Seconds.	
Deg.	Seconds.	Inch.	Parts.	Deg.	Seconds.	Inch.	Parts.
5	1 .7	0	.0016	50	134 .0	0	.1212
10	6 .9	0	.0062	55	153 .2	0	.1386
15	15 .3	0	.0138	60	171 .2	0	.1549
20	26 .7	0	.0246	65	187 .5	0	.1696
25	40 .8	0	.0369	70	201 .6	0	.1824
30	57 .1	0	.0516	75	213 .0	0	.1927
35	75 .1	0	.0679	80	221 .4	0	.2033
40	94 .3	0	.0853	85	226 .5	0	.2050
45	114 .1	0	.1033	90	228 .3	2	.2065

A pendulum that ſwings ſeconds at the equator muſt be $\frac{128}{1000}$ parts of an inch ſhorter, than one that ſwings ſeconds at London; and a pendulum that ſwings ſeconds at the poles, muſt be $\frac{80}{1000}$ parts of an inch longer than one that ſwings ſeconds at London. The cauſe of this difference ariſes from the ſpheroidical

roidical figure of the earth, and the centrifugal force diminiſhing (and ſo acting gradually, leſs and leſs), from the equator to the North and South poles, as the diurnal motions of the places are ſlower and ſlower.

The length of a pendulum that ſwings ſeconds at the equator is 39 inches; and the length of a pendulum that ſwings ſeconds at the poles, is 39.266 inches.

A Deſcription of three uncommon kinds of Clocks.

I. About twenty years ago, I made a wooden model of a clock, which ſhews the apparent diurnal motions of the Sun, Moon, and Stars, with the times of their riſing, ſouthing, and ſetting, for every day of the year; together with all the various phaſes of the Moon, and times of her being New and Full in the different months of the year; with the days of the months, never needing to be ſhifted, ſave once in

in four years; and the age of the Moon for every day of the year, and to every third hour from her change, in any current Lunation. All theſe are ſhewn on the dial-plate, without any confuſion; and I keep the model ſtill by me to ſhew in my lectures.

The outer part of the dial-plate is divided into twice twelve hours, and each hour into eight equal parts, for the half hours, quarters, and half quarters.

Within this circle of hours there is a ring, which goes round once in 24 hours, and carries an index for pointing to the hours of the day and night, and a gilt ball for repreſenting the Sun, and ſhewing his apparent diurnal motion round the earth. This ring is divided into 29 days, 12 hours, and 45 minutes, for the Moon's age from change to change. A ball, half black, half white, is turned rround its axis in 29 days, 12 hours, and 45 minutes, for ſhewing all the various phaſes of the Moon: the axis of the ball lies in the plane of the ring, and comes out a little way

beyond

beyond the Moon, and points to her age in the foreſaid diviſions on the ring, falling back every day as much as the Moon is later of coming to the meridian every day than ſhe was on the day before; and conſequently, the Moon's motion falls back every day ſo in the ring, as to go round it in 29 days, 12 hours, 45 minutes, from the Sun to the Sun again.

Within this ring is a flat circular plate, divided all around its edge into 365 equal parts, for the months and days of the year, which are ſet at the proper diviſions. This plate makes 366 revolutions (as the ſtars do) in the time the Sun makes 365; by which means, the wire that carries the Sun round in 24 hours, cuts the day of the month on the plate, as the plate advances a 365th part of a revolution upon the Sun, once every 24 hours: ſo that the plate turns round in a ſydereal day, which conſiſts of 23 hours, 56 m. 4 ſ. 6 thirds, of mean ſolar time,

and the Sun goes round in 24 mean ſolar hours.

The equator, ecliptic, and tropics are drawn on this plate; the ecliptic is divided into the 12 ſigns of the Zodiac, and each ſign into 30 degrees. The wires, which carry the Sun and Moon, cut their places in the Ecliptic, for every day of the year. All the remarkable ſtars of the firſt and ſecond magnitudes are laid down on this place, according to their right aſcenſions and declinations.

Over this plate is a fixt horizon for ſhewing the times of the riſing and ſetting of the Sun, Moon, and Stars. When any ſtar comes to the Eaſt ſide of the horizon, it riſes; and the hour-hand points to the time of its riſing: when it ſets on the Weſtern ſide of the horizon, the hour-hand points out the time of its ſetting; on any day of the year which the Sun's wire then cuts.

When the points of the ecliptic, which are cut by the Sun's wire, and the Moon's, come to the Eaſt and Weſt

ſides

ſides of the horizon, the hour-hand points to the times of their riſing and ſetting, on the day of the year which the wire then cuts that carries the Sun.

The wheel-work of this clock is as follows.

In the center, behind the middle of the Dial-plate, is a fixt pinion of 16 leaves, round which one wheel of 100 teeth, and another of 70 are carried, every 24 hours; the leaves of the pinion taking into the teeth of the wheels. On the axis of the wheel of 100 teeth is a pinion of 14 leaves, which turns a wheel of 69 teeth, on whoſe axis is a pinion of 7 leaves, turning a wheel of 83 teeth, which wheel is pinned to the back of the ſydereal flat plate above-mentioned, which has the months and ſigns &c. upon it; and any given point in the edge of this laſt wheel and plate, revolves from the meridian to the meridian again, in 23 hours, 56 min. 4 ſec. 6 thirds (which makes a ſydereal day) and from the Sun to the Sun again (which revolves in 24 hours),

 in

in 365 days, 5 hours, 48 minutes, 58 ſeconds, and 47 thirds, of mean ſolar time.

The above-mentioned wheel of 70 teeth (which is carried round the fixt pinion of 16 leaves every 24 hours) has a pinion of 8 leaves on its axis, which turns a wheel of 54 teeth; and to the axis of this wheel of 54 the Moon's wire is fixed, which carries the Moon round, from the meridian to the meridian again, in 24 hours*, 50 min. 25 ſeconds; round the ecliptic on the flat plate in 27 days, 7 hours, 43 minutes, and round from the Sun to the Sun again (or from change to change) in 29 days, 12 hours, 45 minutes.

II. About ten years ago, I made a wooden model of a clock for ſhewing the apparent diurnal motions of the

* It is generally believed that the Moon revolves from the meridian to the meridian again, in 24 h. 48 min. but that is a miſtake; for if ſhe did, there would be 30 compleat days from change to change.

Sun

Sun and Stars, with the times of their riſing and ſetting for every day of the year; and the days of the months all the year round, without any need of ſhifting by hand in the ſhort months, as is always done in common clocks. I copied the Dial-plate of this model from a clock that Mr. Ellicott had made for the king of Spain: but although Mr. Ellicott ſhewed me the whole in-ſide of the clock, I did not aſk him what the numbers of teeth in the wheels of it were, although, I am convinced, he would have told me, if I had; nor do I, in the leaſt, remember how many wheels there were in the uncommon or Aſtronomical part of it; and ſo I ſet about contriving wheels and numbers for performing the like motions.

The Dial-plate contains twice twelve hours, and within the circle of hours there is a large opening in the plate, a little elliptical: the edge of this opening ſerves for an horizon.

Below the Dial-plate, and ſeen through the large opening in it, is a flat

ſlat plate on which the equator, ecliptic, and tropics, are drawn; and all the ſtars of the firſt, ſecond, and third magnitudes are laid down, that are viſible in the horizon (of Madrid in Mr. Ellicott's, and of London in mine) according to their right aſcenſions and declinations: the center of the plate being the North pole. The ecliptic is cut out into a narrow groove in the plate; and a ſmall Sun ſlides in the groove by a pin, and is carried round by a wire fixt in the axis, which comes a little way out through the center of the plate. The edge of this plate is divided into the months and days of the year, and the Sun's wire ſhews the days of the months in theſe diviſions. This ſtar plate goes round in a ſydereal day, making 366 revolutions in a year; in which time the Sun makes 365, and conſequently ſhifts a diviſion, or day of the month, every 24 hours.

A ſmall wire is ſtretched from over the center of the ſydereal plate to the

upper

upper XII on the fixed Dial-plate. This wire is for the meridian.

When the Sun, or any ſtar, comes to the Eaſtern edge of the horizon, the hour index is at the time of riſing of the Sun or Star, for the day of the year, pointed to by the wire, that carries the Sun: and when the Sun or Star comes to the Weſtern edge of the horizon, the hour index is at the time of its ſetting. The Sun always comes to the meridian at the inſtant of the ſolar noon; but every ſtar comes ſooner to the meridian every day, than it did on the day before, by 3 minutes, 55 ſeconds, 54 thirds of mean ſolar time, as it revolves from the meridian to the meridian again in 23 hours, 56 m. 4 ſ. 6 thirds.

When any ſtar is on the meridian in the clock, the ſtar which it repreſents is on the meridian in the heavens; the time whereof is ſeen by the hour index on the Dial-plate. And, as the ſtars have their revolutions on the plate, one may look at the clock at any time, and ſee

ſee what ſtars are then above the horizon, what ſtars are then on the meridian, and what ſtars are then riſing and ſetting.

My contrivance for ſhewing theſe motions and phenomena, in the model, conſiſts of no more than two wheels and two pinions, as follows.

The wheels are of equal diameters, and ſo are the pinions; the numbers of teeth are 61 in one wheel, and 73 in the other. The pinions are both fixt on one axis, the one having 20 leaves and the other 24. The wheel of 73 teeth is fixed to the back of the ſydereal wheel, and the axis of the wheel of 61 comes through the wheel of 73, and through the ſydereal plate, and carries the wire round on which the Sun ſides in the ecliptic groove, and alſo the hour hand on the Dial-plate.

The wheel of 61 teeth turns the pinion of 20, and the pinion of 24, (fixt on the ſame axis with that of 20) turns the wheel of 73.

Now, if the wheel of 61 teeth be turned round in 24 hours, to carry the

Sun and hour hand, the wheel of 73 teeth will be turned round in 23 hours, 56 minutes, 4 ſeconds, 6 thirds. And ſo, the ſydereal plate will make juſt 366 revolutions, in the time that the Sun makes 365.

Mr. Ellicott had the prettieſt, and moſt ſimple contrivance I ever ſaw, in his clock, for ſhewing the difference between equal and ſolar time (generally termed the equation time) on all the different days of the year. He generouſly allowed me to copy that part into my model, and I have quite concealed it within one of my wheels, not to ſhew how it is done unleſs he publiſhes an account of it. The Sun, by that ſimple contrivance, even in my model, comes as much ſooner or later to the meridian, than when it is Noon by a well-regulated clock, as the Sun in the heavens does, at all the different times of the year, excepting the four days on which the time of Noon ſhewn by the Sun and clock ought to coincide: and then there is no difference in

the clock. And although the wheel work is quite open to ſight in the model which I now ſhew in my lectures, no perſon who ſees it can gueſs how the unequal motion of the Sun, in the model, is performed.

III. In the year 1764, when I happened to be at Liverpool, I contrived a clock for Captain Hutchinſon, who is Dock-maſter of the place, for ſhewing the age and phaſes of the Moon, and the time of High and Low water at Liverpool, every day of the year, with the ſtate of the tides at any time of the day; by looking at the clock.

At the right and left lower corners of the Dial-plate, under the common circles for the hours and minutes, there are two ſmall circular plates. On the plate at the left hand there are two circular ſpaces, the outermoſt of which is divided into twice twelve hours, with their halves and quarters: within which, the ſecond circular ſpace is divided into $29\frac{1}{2}$ equal parts for the days of the Moon's

Moon's age; each day ſtanding under the time of the Moon's coming to the meridian on that day, in the circle of 24 hours. An axis comes through the center of this plate, and carries two indexes round it in 29 days, 12 h. 45 min. or from change to change of the moon: and theſe indexes are ſet as far aſunder, as the time of High water at Liverpool differs from the time of the Moon's coming to the meridian. So that, by looking on this plate in the Morning, one may ſee at what time the Moon will be on the meridian, and at what time it will be High water at the place.

On the right hand plate, around its edge, all the different ſtates of the tides are marked, from High to Low, and from Low to High; and within theſe appellations is a ſhaded ellipſis, the higheſt points of which repreſent High water, and the loweſt parts Low water. An index goes round this plate in the time of the Moon's revolving from the meridian to the meridian again; and,

at all different times, points out the ſtate of the tide, as it may be then High or Low, riſing or falling.

In the arch of the Dial-plate above the hour of XII, a blue plate riſes and falls as the Tides do at Liverpool: and, over this plate, in a painted ſky, a globular ball, half black, half white, ſhews the phaſes of the Moon for every day of her age, throughout the year.

The wheel-work for ſhewing theſe appearances is as follows.

A wheel of 30 teeth is fixed on the axis of the twelve-hour hand, and turns round with it. This wheel turns a wheel of 60 teeth, round in 24 hours, and on its axis is a wheel of 57 teeth, which turns round in the ſame time, and turns a wheel of 59 teeth round in 24 hours 50½ minutes, on whoſe axis is the index on the right hand corner-plate, going round the plate in the time of the Moon's revolving from the meridian to the meridian again; and ſhewing the ſtate of the Tide at any time, when the clock is looked

looked at. On the axis of the ſame wheel of 59 teeth is fixed an elliptical plate, which raiſes and lets down the Tide-plate in the arch, twice in 24 hours $50\frac{1}{2}$ minutes; in which time there are two ebbings and flowings of the Tides.

The above-mentioned wheel of 57 teeth has a pinion of 16 leaves on its axis, turning a wheel of 70 teeth, on whoſe axis is a pinion of 8 leaves, turning a wheel of 40 teeth, which turns a wheel of 54 teeth round in 29 days, 12 hours, 45 minutes; and on the axis of the wheel of 54 teeth are the two indexes on the left hand corner plate, for ſhewing the Moon's age on that plate, with the time of her ſouthing, and of High water.

The wheel of 40 teeth here mentioned, might have been of any other number, and might have been left out, if the pinion of 8 leaves had taken into the wheel of 54 teeth; but then the index would have gone the wrong way round the dial plate. So that the only

ufe of the wheel of 40 is to be a leading wheel, for turning the index round the right way.

On the axis of the wheel of 54 teeth (which turns round in a lunation) is a fmall wheel of 20, turning a contrate wheel of the fame number, on whofe axis is the globular Moon (half black, half white) in the arch, turning round in a lunation, and fhewing all her phafes.

In thefe three clocks, I have only defcribed the uncommon parts, which are connected with the common part of the movement known unto every Clock-maker.

An eafy way of reprefenting the apparent diurnal motions of the Sun and Moon in a clock.

Let a thick wheel of 57 teeth be turned round in 24 hours, and take into the teeth of two wheels of equal diameters, one of which has 57 teeth and the other 59; thefe wheels lying clofe

cloſe upon one another, and the axis of the one turning within the axis of the other. A wire fixt on the axis of the wheel of 57 will carry a Sun round in 24 hours; and a wire fixt on the axis of the wheel of 59 will carry a Moon round in 24 hours $50\frac{1}{2}$ minutes. If the Sun carries a plate round with him in 24 hours, and the limb of the plate be divided into $29\frac{1}{2}$ equal parts for the days of the Moon's age, the Moon will ſhew her age in the diviſions of that plate; and may be made to turn round her axis, and ſhew her phaſes, by a wheel of any number of teeth, on her axis, and taking into the teeth of a contrate wheel of the ſame number, fixt on the axis of the wheel of 57 teeth, which carries the Sun.

An eaſy way of ſhewing the phaſes of the Moon, in a Clock.

Let a wheel of 16 teeth be fixed on the axis of a wheel of 15, and the wheel of 16 turn a wheel of 63, on whoſe

whoſe axis let a ball, half black, half white, be fixed; and project half way out, through a round hole in the dial plate.

Then, if the wheel of 15 teeth be always moved one tooth in 12 hours, the ball will be turned round in 29 days, 12 hours, 45 minutes, and ſhew all the various phaſes of the Moon.

An eaſy method of ſhewing the Sun's place in the Ecliptic every day of the year, in a clock; and his motion round the Ecliptic in a Solar year.

Let a pinion of 12 leaves be turned round once every ten hours, and this pinion take into a wheel of 67 teeth, on whoſe axis let there be a ſingle threaded ſcrew taking into a wheel of 157 teeth. This laſt wheel will turn round in 365 days, 5 h. 49 m. 50 ſec. And an index on its axis will carry a Sun through the whole 360 degrees of an ecliptic, engraven on the dial plate in the ſame time: and may ſhew the

the days of the months on another circle within the ecliptic. This was the contrivance of Mr. *Arnſhaw*, near *Mancheſter*, who communicated it to me.

How to ſhew the periodical revolutions of the Earth, and all the other planets, round the Sun, in a Clock; ſo as to agree nearly with the periodical revolutions of the planets about the Sun in the Heavens.

Let ſix hollow ſockets, or arbors, be made to fit and turn within one another, and all of them to turn upon a fixt ſpindle, or axis; on the top of which let there be a ball to repreſent the Sun. Let the wideſt arbor be the ſhorteſt, and have an arm on its uppermoſt end to carry a ball repreſenting Saturn, and a wheel of 206 teeth on its lowermoſt end.

Let the next ſized arbor be ſo much longer than the above one, as to have a wheel (of 83 teeth) put upon it, below the wheel of 206; and an arm on

the other end (above Saturn's) for carrying a ball to repreſent Jupiter.

Let the third ſocket be ſo much longer than the ſecond, as to have a wheel on it (of 47 teeth) below the wheel of 206, and an arm on its other end, above Jupiter's, for carrying a ball to repreſent Mars.

Let the fourth arbor be ſo much longer than the third, as to have a wheel (of 40 teeth) on its lower end, and an arm on its upper end, above Mars's, for carrying a ball to repreſent the Earth.

Let the fifth arbor be ſo much longer than the fourth, as to have a wheel (of 32 teeth) on its lower end, below the wheel of 40, and an arm on its upper end, above the Earth's, for carrying a ball to repreſent the planet Venus.

Let the ſixth (which is the ſmalleſt) arbor, be ſo much longer than the fifth, as to have a wheel (of 20 teeth) on its lower end, below the wheel of 32, and an arm on its upper end, above

Venus's,

Venus's, for carrying a ball to repreſent the planet Mercury.

Saturn's arm muſt be the longeſt of all, becauſe that planet is the furtheſt of all from the Sun: Jupiter's the next longeſt, Mars's the next, the Earth's the next, Venus's the next, and Mercury's the next, or ſhorteſt of all, becauſe Mercury is the neareſt of all the planets to the Sun.

The wheels muſt be fixed on their reſpective arbors, and diminiſh in their ſizes from the higheſt numbers to the loweſt; ſo that, when they are all put together, they may form ſomewhat of the appearance of a cone.

And, to give theſe wheels and planets their proper motions, they muſt be turned by ſix wheels (or rather four wheels and two pinions) all fixed on one ſolid axis, in a conical manner, inverted with reſpect to the other ſix wheels; ſo as the wheels and pinions on the ſolid axis may take into thoſe on the arbors, and turn them.

The ſolid axis, with all its wheels and pinions, will turn round in the ſame time together, becauſe the wheels and pinions are all fixed on the axis; and muſt be turned round once in a year by the clock-work; which may be eaſily done by ſuch a method as Mr. *Arnſhaw's*, already mentioned.

Then, if the uppermoſt and ſmalleſt pinion on the axis has 7 leaves, taking into Saturn's wheel of 206 teeth; Saturn will be carried round the Sun in 10748 days, 18 hours, 43 minutes: for, as 7 is to 206, ſo is 365.25 to 10748.78.

If the next pinion on the axis (which muſt be of a bigger ſize than the pinion of 7 above it) has 7 leaves, and takes into Jupiter's wheel of 83 teeth; Jupiter will be carried round the Sun in 4330 days, 19 hours, 40 minutes: for, as 7 is to 83, ſo is 365.25 to 4330.82.

If the wheel below this pinion on the axis has 25 teeth, and takes into Mars's wheel of 47; Mars will be carried round the Sun in 686 days, 16 hours,

hours, 5 minutes: for, as 25 is to 47, ſo is 365.25 to 686.67.

If the next bigger wheel on the axis, which turns round in 365.25 days, has 40 teeth, and takes into the Earth's wheel of 40 teeth; the Earth will be carried round the Sun in 365.25 days.

If the next bigger wheel has 52 teeth, and takes into Venus's wheel of 32 teeth; Venus will be carried round the Sun in 224 days, 18 hours, 29 minutes: for, as 52 is to 32, ſo is 365.25 to 224.77.

And laſtly, if the largeſt wheel on the axis has 83 teeth, and takes into Mercury's wheel of 20; Mercury will be carried round the Sun in 88 days, 0 hours, 14 minutes: for, as 83 is to 20, ſo is 365.25 to 88.01.

I have ſeen a calculation of this ſort in a printed book; but the numbers there are ſo faulty for Mars and Saturn, that I was obliged to alter them; Saturn's period being wrong by 51 days. How near theſe are to the truth will appear by comparing them with the annual periods in the following Table.

A Table

A Table ſhewing the times contained in the annual and diurnal revolutions of the Planets; with their relative and true diſtances from the Sun, the circumferences of their Orbits, the number of miles they move every hour therein, and their daily mean motions in degrees and parts of a degree. Their diſtances in miles from the Sun are here ſet down, as they were found to be by the Tranſit of Venus over the Sun, June 6, 1761.

The fix primary Planets.	Their annual Periods round the Sun.	Their diurnal Rotations.	Hourly motion of their Equators.	Their relative mean diſt. fr the Sun	Their real diſtances from the Sun in Engliſh miles.	The circumferences of their Orbits in Engliſh miles.	The number of miles they move in each hour, in their Orbits.	Their daily mean motions in their Orbits.
	d. h.		Miles.	Parts.			Eng. Miles.	° ′ ″
Mercury	87 23	Unknown.	Unknown.	3871	36,841,468	231,574,940	109699.16	4 5 32
Venus	224 17	24d. 8h.	43	7233	68,891,486	433,032,198	80295.24	1 36 8
The Earth	365 6	1d. 0h.	1042	10000	95,173,000	598,230,286	68243.24	0 59 8
Mars	686 23	24h. 40m.	556	15237	145,014,148	911,517,502	55287.00	0 31 27
Jupiter	4332 12	9h. 56m.	25920	52009	494,990,976	3,111,371,849	29083.60	0 4 59
Saturn	10759 7	Unknown.	Unknown.	95400	907,956,130	5,707,152,817	22101.64	0 2 1

A PROBLEM.

A PROBLEM.

I.

Suppoſe there are ſix hands on the Dial-plate of a clock, all going round the ſame way; and that the firſt, or ſloweſt hand, A, *goes round in* 24 *hours; the next ſloweſt hand,* B, *in* 22 *hours; the next,* C, *in* 20 *hours; the next,* D, *in* 18; *the next,* E, *in* 16; *and the laſt, or ſwifteſt,* F, *in* 14 *hours: and that they all ſet off together, from a conjunction, at any given point of the Dial-plate.* Qu. *in how many hours afterward will they all be in conjunction again, and how many revolutions will each hand have made in that time?*

Let a, b, c, d, e, f, be the periodical times or revolutions of A, B, C, D, E, F, then, a, will be 24 hours, b 22, c 20, d 18, e 16, and f 14.

I. The canon for finding the time that muſt elapſe between the con-

junctions

junctions of *A* and *B*, is $\frac{ab}{a-b}$, and consequently all its multiples; *viz.* $\frac{2ab}{a-b}$, $\frac{3ab}{a-b}$, $\frac{4ab}{a-b}$ &c. on to $\frac{mab}{a-b}$; where *m* is any indefinite number of conjunctions.

For, if *A* and *B* are in conjunction at the end of the time $\frac{ab}{a-b}$, 'tis evident they will be in conjunction again when as much more time has elapsed; and so on to infinity.

2. The canon for finding the times between the conjunctions of *B* and *C* is $\frac{bc}{b-c}$ and all its multiples indefinitely; as $\frac{2bc}{b-c}$, $\frac{3bc}{b-c}$, &c. to $\frac{nbc}{b-c}$.

And therefore, when $\frac{ab}{a-b}$, or its multiple, is equal to $\frac{bc}{b-c}$ or its multiple, *A*, *B*, and *C* will then be in conjunction again. For, by the first expression, *A* and *B* will be in conjunction; and by the second, *B* and *C* will be so too. But the expressions being equal, the times must also be equal: that is, *A*, *B*, and *C* will be in conjunction again.

3. The canon for finding the times between the conjunctions of *C* and *D* is $\frac{cd}{c-d}$, and all its multiples indefinitely,

as

as above. And therefore, when any multiple of the conjunctions of *A*, *B*, and *C*, is equal to any multiple of the conjunctions of *C* and *D*, then *A*, *B*, *C*, and *D*, will be in conjunction again.

4. The canon for finding the times between the conjunctions of *D* and *E* is $\frac{de}{d-e}$, and all its multiples; and therefore when any multiple of the conjunctions of *A*, *B*, *C*, and *D*, is equal to any multiple of the conjunctions of *D* and *E*, then, *A*, *B*, *C*, *D*, and *E*, will be in conjunction again.

5. The canon for finding the times between the conjunctions of *E*, and *F*, is $\frac{ef}{e-f}$, and all its multiples indefinitely; and therefore, when any multiple of the conjunctions of *A*, *B*, *C*, *D* and *E*, is equal to any multiple of the conjunctions of *E* and *F*, all the hands *A*, *B*, *C*, *D*, *E*, and *F*, will be again in conjunction.

The multiples must all be whole numbers, and the least that will do must be taken, to find the times between the next succeeding conjunctions.

The times between the conjunctions of theſe ſix hands, taking them by two and two, are as follows,

$\frac{ab}{a-b}=264$; the number of hours in which *A* and *B* will come to their next conjunction, after their firſt ſetting out together.

$\frac{bc}{b-c}=220$; the number of hours in which *B* and *C* will come to their next conjunction, after their firſt ſetting out together.

$\frac{cd}{c-d}=180$; the number of hours in which *C* and *D* will come to their next conjunction, after their firſt ſetting out together.

$\frac{de}{d-e}=144$; the number of hours in which *D* and *E* will come to their next conjunction, after their firſt ſetting out together. And.

$\frac{ef}{e-f}=112$; the number of hours in which *E* and *F* will come to their next conjunction, after their firſt ſetting out together.

In working for ſuch multiples (of integer numbers) as will make the above

above expreſſions equal, in the leaſt ratios of the times, I find they are as follows:

264 (or $\frac{ab}{a-b}$) multiplied by 5, is equal to 220 (or $\frac{bc}{b-c}$) multiplied by 6; equal to 1320 hours, for the time in which *A*, *B*, and *C*, will come to their next conjunction, after their firſt ſetting out together. And

1320 hours (the conjunction of *A*, *B*, and *C*) multiplied by 3, is equal to 180 hours (or $\frac{cd}{c-d}$) multiplied by 22; equal to 3960 hours, for the time of the next conjunction of *A*, *B*, *C*, and *D*, after their firſt ſetting out together. And

3960 hours (the conjunction of *A*, *B*, *C*, and *D*) multiplied by 4, is equal to 144 hours (or $\frac{de}{d-e}$) multiplied by 110; equal to 15840 hours, for the time of the next conjunction of *A*, *B*, *C*, *D*, and *E*, after their firſt ſetting out together. And

15840 hours (the laſt mentioned conjunction) multiplied by 7, is equal

 to

to 112 hours (the conjunction of *E* and *F*, or $\frac{ef}{e-f}$) multiplied by 990, equal to 110880 hours, the time when all the six hands, *A*, *B*, *C*, *D*, *E*, and *F*, will be in conjunction again, after the instant of their first setting out together, from a conjunction at any given point of the Dial-plate; and all moving round it the same way, in the times above-mentioned.

Now, as it will require 110880 hours) (or 4260 days) to bring all these hands together again, after their first setting out together; divide 110880 by 24 hours, the time of *A*'s going round; by 22, the time of *B*'s going round; by 20, the time of *C*'s; by 18, the time of *D's*; by 16, the time of *E's*; and by 14 hours, the time of *F*'s going round: and the quotients will shew that *A* has made 4260 revolutions, *B* 5040, *C* 5544, *D* 6160, *E* 6930, and *F* 7920. And, at the end of so many more revolutions of each hand, they will all be in conjunction again; and so on continually.

II.

II.

The periodical times of the six primary planets being given, and supposing them to have been all at once in a line of conjunction with the Sun; to find how much time would elapse before they were all in a line of conjunction with the Sun again. This Problem I had from Mr. *Waring* (now Professor of Mathematics in the University of *Cambridge)* in the year 1755.

Let a, b, c, d, e, f, be respectively equal to the periodical times or revolutions of the six planets about the Sun; a being the longest, or Saturn's period; b the next longest, or Jupiter's; c the next, or Mars's; d the next, or the Earth's; e the next, or Venus's; and f the shortest period of all, which is Mercury's: and let p, qp, rqp, $srqp$, &c. be equal to the difference or time between the succeeding conjunctions of any two, three, four, &c. of them. 'Tis

'Tis evident that q, r, s (the multiples) muſt be whole numbers, becauſe the numbers of conjunctions are ſo.

The time between the conjunctions of the firſt two is $\frac{ab}{a-b}, =p$; that of the firſt three is $\frac{n \times ac}{a-c}, =qp$ (where n is any number aſſumed, to make q a whole number) or, which is the ſame, $\frac{nac}{\overline{a-c} \times p}=q$; $\frac{ac}{\overline{a-c} \times p}$ being reduced to its loweſt denominator, q will be equal to the numerator of that fraction. In the ſame manner, r will be equal to the numerator of the fraction $\frac{ad}{\overline{a-d} \times qp}$ reduced to its loweſt denominator; s will be equal to the numerator of the fraction $\frac{ae}{\overline{a-e} \times rqp}$, reduced to its loweſt denominator; and ſo on, from the ſloweſt to the quickeſt revolving bodies in the ſyſtem: by which means, the times of all their conjunctions may be found.

This

This Problem may be solved by a different method, as follows; for which I am obliged to my generous friend Mr. John Ford, *Surgeon in Bristol.*

Let A, B, C, D, E, F, stand for the six planets, beginning with Saturn, and ending with Mercury; and a, b, c, d, e, f, be the times of their periodical revolutions respectively. Then, by a known rule, the synodical period, or conjunction, of A and B, will be the time $\frac{ab}{a-b}$; and that of B and C will be $\frac{bc}{b-c}$; that of C and D will be $\frac{cd}{c-d}$; that of D and E will be $\frac{de}{d-e}$; and that of E and F will be $\frac{ef}{e-f}$.

Now it is obvious, that A and B can never be in conjunction but in the time $\frac{ab}{a-b}$, or some multiple of it; neither can B and C be in conjunction but in the time $\frac{bc}{b-c}$, or some multiple of that time. A, B, and C, will therefore be in conjunction when $\frac{mab}{a-b}$ is equal to $\frac{nbc}{b-c}$, where m and n represent two integer numbers, prime to each other; which being re-

spectively

ſpectively multiplied into $\frac{ab}{a-b}$ and $\frac{bc}{b-c}$ ſhall make the two products equal. And theſe two numbers are eaſily diſcovered; for, as by ſuppoſition, $\frac{mab}{a-b}$ is equal to $\frac{nbc}{b-c}$, therefore, $m.n :: \frac{bc}{b-c} : \frac{ab}{a-b}$. Reduce therefore $\frac{bc}{b-c}$ and $\frac{ab}{a-b}$ into integers of the leaſt dimenſions (as minutes, or ſeconds of time) which ſhall have the ſame proportion to each other as theſe numbers have; and you will have the multiples m and n, and conſequently the ſynodical period or conjunction of A, B, and C; which we ſhall call R. In the ſame manner may the ſynodical period of C, D, and E, be inveſtigated, which call S: then find two prime numbers r and s in their loweſt dimenſions, which ſhall have the ſame proportion to each other as the times R and S; then will rS, or its equal Sr, give the ſynodical period, or conjunction of the five planets, A, B, C, D, and E, which characterize by T. Find laſtly, the ſynodical period of E and F, by the rule $\frac{ef}{e-f}$, which denote by X; and the leaſt integer numbers

bers t, x, in the ſame proportion to each other as T and X being found, tX or xT will be the ſynodical period, or conjunction, of the ſix primary planets, A, B, C, D, E, F; or the time that muſt elapſe between any conjunction of them all, and the next ſucceeding conjunction. Which time, being divided by the time of the periodical revolution of each planet, will ſhew how many revolutions each planet has then made.

There are ſeveral ways of finding the above-mentioned prime integer numbers or multipliers; but the following is very convenient and eaſy.

Let $\frac{a}{b}$ and $\frac{c}{d}$ be two of the fractions. Multiply the denominator of the firſt into the numerator of the ſecond, and *vice verſâ*; then ſtrike out both the denominators, by which proceſs the above fractions become ad and bc; which numbers are in the ſame proportion as the fractions; and, if they are prime to each other, are the numbers required. But if they are not

 prime,

prime, divide them by their greateſt common diviſor, in order to reduce them to their loweſt denomination.

The reaſon why theſe numbers muſt be prime integers is plain: for, if they were not ſo, we ſhould not have the ſynodic period required, but ſome multiple of it: and if they were not integers, we ſhonld not have exact multiples of the lower ſynodic periods from which we deduce the higher.

To facilitate calculations which may be made on theſe principles, I ſhall ſubjoin the following Table, which ſhews the annual periods of the primary planets, reduced to hours; and their ſynodical periods, taken two by two progreſſively. But although the ſynodical periods of the planets, taken two by two, is ſo ſhort, it muſt not be imagined that the ſynodical periods of three planets muſt be proportionably ſo too. The ſynodic period of the Earth and Venus (by the Table) is 1 year, 218 days, 17 hours; and that of Venus and Mercury is 144 days, 12 hours; but

but the ſynodical period of theſe three planets is upwards of 5500 years.

If the periods of three planets be ſo incommenſurate, how much more ſo muſt be the periods of the ſix revolving primaries of our ſyſtem? Indeed we here cannot but ſee and admire the wiſdom and providence of the SUPREME BEING! For, had the times of the annual revolutions of the ſeveral planets been more commenſurate, the preſent arrangement of our ſyſtem would doubtleſs have been greatly diſturbed by the conſpiring attraction of the ſix bodies, when they happened to be in conjunction; an arrangement, which, from the goodneſs of the Almighty, we muſt conclude to be, in its preſent ſtate, the beſt adapted to anſwer the purpoſes for which the ſyſtem was created.

Names of the Planets.	Their periodical revolutions reduced to hours.	
Saturn	258223^h.	$= a$
Jupiter	103980	$= b$
Mars	16487	$= c$
Earth	8766	$= d$
Venus	5393	$= e$
Mercury	2111	$= f$

Their ſynodical periods, or conjunctions with each other.

	Y.	D.	H.	Hours.	
Saturn and Jupiter	19	313	10 =	174076 =	$\frac{ab}{a-b}$
Jupiter and Mars	2	85	21 =	19593 =	$\frac{bc}{b-c}$
Mars and the Earth	2	49	10 =	18718 =	$\frac{cd}{c-d}$
Earth and Venus	1	218	17 =	14015 =	$\frac{de}{d-e}$
Venus and Mercury	0	144	12 =	3468 =	$\frac{ef}{e-f}$

To illuſtrate the uſe of this Table, let it be required to find the ſynodical period or conjunction of the Earth, Venus, and Mercury.

That of the Earth and Venus 14015 hours, $=\frac{de}{d-e}$; and that of Venus and Mercury is 3468 hours, $=\frac{ef}{e-f}$.

Therefore, from what has been already laid down (ſee pag. 143) the ſynodical period of the three planets will be when $m \times 14015$ is equal to $n \times 3468$; or when $m : n :: 3468 : 14015$; m and n being the leaſt integer numbers in the proportion of 3468 to 14015. But theſe numbers being integers, and in

in their loweſt terms already, they require no reduction. Therefore, 3468 × 14015 give the ſynodical period of the three planets, = 48604020 hours; = 5544 years, 221 days, 12 hours. The reader may proceed to find out the ſynodic periods or conjunctions of the reſt, according to the foregoing rules.

The following problem is of the ſame nature with this; but, as it is more familiar and obvious, it may better ſerve to confirm the truth of the method we have uſed to inveſtigate the ſynodical period of bodies revolving the ſame way, but in different times, about the ſame common center.

III.

Suppoſe the hour, minute, and ſecond-hands of a Clock to be in conjunction at the hour of XII. *It is required to find when they will be in conjunction again?*

Here we have the periodical revolution of the hour had = 720 minutes, $= a$; the periodical revolution of the minute-

minute-hand $= 60$ minutes, $= b$; and *that* of the second-hand $= 1$ minute, $= c$: from whence we collect $\frac{ab}{a-b}$, $= \frac{43200}{660}$ min. $= \frac{7200}{110} = \frac{720}{11}$ min. for the synodical period or conjunction of the hour and minute hands; and $\frac{bc}{b-c} = \frac{60}{59}$ for the synodical period of the minute and second hands. Then, to find the synodical period of all the three hands, we must (as in the above Problem) suppose $\frac{m \times 720}{11} = \frac{n \times 60}{59}$; from whence we have $m : n :: \frac{60}{59} : \frac{720}{11}$. Now, the least integer numbers, represented by m and n, in the proportion of $\frac{60}{59}$ to $\frac{720}{11}$ are 11 and 708. Therefore $11 : 708 :: \frac{60}{59} : \frac{720}{11}$; and consequently $\frac{11 \times 720}{11}$ $(= 708 \times \frac{60}{59})$ = the synodical period of the three hands of the Clock; $= 720$ minutes, or just 12 hours.

The periodical revolutions of the Sun and Moon, round the Ecliptic, and their synodical periods or conjunctions with each other, may be familiarly represented by the motions of the hour and

and minute hands of a watch, round its Dial-plate. For, the Dial-plate is divided into 12 hours, as the Ecliptic is divided into 12 ſigns; the hour-hand goes round in 12 hours, as the Sun does in 12 months, and the minute hand goes round in 1 hour, as the Moon does in (ſomewhat leſs than) a month. And, as the Moon never is in conjunction with the Sun in that point of the Ecliptic where ſhe was at the laſt conjunction before, ſo the minute-hand never is in conjunction with the hour-hand at that point of the Dial-plate where it was at the laſt preceding conjunction. So that, the 12 hours on the Dial-plate may repreſent the 12 ſigns of the Ecliptic; the hour-hand the Sun, and the minute-hand the Moon: only, the motion of the minute-hand is too ſlow for the Moon in proportion to *that* of the hour-hand compared with the motion of the Sun. For, in the time of the Sun's going round the Ecliptic, which is 12 calendar months, there are 12.36 conjunctions of the

Sun

Sun and Moon; but in the time the hour-hand goes round the Dial-plate, the minute-hand is only 11 times conjoined with it.

Theſe hands are always in conjunction at XII o'clock. The firſt column of the Table ſhews the number of their conjunctions in 12 hours, and the collateral lines ſhew in how many hours, minutes &c. after XII, they come to their ſucceeding conjunctions marked in the firſt column; the time between any conjunction and the next being 1 hour, $5\frac{5}{11}$ miuutes.

Conj.	Hou.	m.	II	III	IV	V	VI	VII	VIII	IX	X
1	I	5	27	16	21	49	5	27	16	21	$49\frac{1}{11}$
2	II	10	54	32	43	38	10	54	32	43	$38\frac{2}{11}$
3	III	16	21	49	5	27	16	21	49	5	$27\frac{3}{11}$
4	IIII	21	49	5	27	16	21	49	5	27	$16\frac{4}{11}$
5	V	27	16	21	49	5	27	16	21	49	$5\frac{5}{11}$
6	VI	32	43	38	10	54	32	43	38	10	$54\frac{6}{11}$
7	VII	38	10	54	32	43	38	10	54	32	$43\frac{7}{11}$
8	VIII	43	38	10	54	32	43	38	10	54	$32\frac{8}{11}$
9	IX	49	5	27	16	21	49	5	27	16	$21\frac{9}{11}$
10	X	54	32	43	38	10	54	32	43	38	$10\frac{10}{11}$
11	XII	0	0	0	0	0	0	0	0	0	0

If the above proceſs was carried on to infinity, in the horizontal lines, the numbers would circulate at every fifth column.

To aſſiſt the imagination in forming an idea of the vaſt diſtances of the planets from the Sun, let us ſuppoſe, that a body projected from the Sun, ſhould continue to fly with the ſwiftneſs of a cannon ball, *viz.* 480 miles every hour; this body would reach the orbit of Mercury in 8 Julian years, 276 days; of Venus, in 16 years, 136 days; of the Earth, in 22 years, 226 days; of Mars, in 34 years, 170 days; of Jupiter, in 117 years, 234 days; and the orbit of Saturn, in 215 years and 286 days.

If the reader ſhould think this idea too extenſive (notwithſtanding its being a juſt one) he may contract it in the following manner, which takes in both the proportional bulks and diſtances of the Sun and Planets.

The Dome of St. Paul's is 145 feet in diameter. Suppoſe a globe of this ſize to repreſent the Sun: then, a globe of $9\frac{7}{10}$ inches will repreſent Mercury; one of $17\frac{9}{10}$ inches, Venus; one of 18 inches, the Earth; one of 5 inches diameter, the Moon (whoſe diſtance from

the Earth is 240000 miles) one of 10 inches, Mars; one of 15 feet, Jupiter; and one of 11½ feet, Saturn, with his ring four feet broad, and at the ſame diſtance from his body, all around.

In this proportion, ſuppoſe the Sun to be at St. Paul's; then Mercury might be at the Tower of London; Venus at St. James's Palace; the Earth at Marybone; Mars at Kenſington; Jupiter at Hampton-Court, and Saturn at Cliefden: all moving round the Cupola of St. Paul's as their common center.

To repreſent the motions of Jupiter's four Satellites round Jupiter, in a clock; and ſhew the times of their Eclipſes in Jupiter's ſhadow.

On four hollow arbors, let there be four bent wires of different lengths, to carry the Satellites round Jupiter, as the arbors are turned round within one another; and let Jupiter be fixed on the top of a ſolid axis or ſpindle, on which all the arbors are turned round;

the wires being ſo bent, as that the Satellites, on their tops, may be of the ſame height with Jupiter's ball. The diameters of the Satellites ſhould not be above a ſixth or ſeventh part of the diameter of Jupiter; and, to be at their proper diſtances from him, the diſtance of the neareſt Satellite ſhould be $5\frac{2}{3}$ ſemidiameters of Jupiter from his center; the ſecond Satellite, 9 ſemidiameters of Jupiter diſtant from his center; the third $14\frac{1}{3}$ ſemidiameters; and the fourth, $25\frac{1}{3}$ of his ſemidiameters from his center.

Let four wheels of different ſizes, and different numbers of teeth, be fixed upon the lower end of the abovementioned arbors, in a conical manner, as deſcribed in the former machine (pag. 129); the wheel on the ſmalleſt arbor, that carries the firſt Satellite, having 22 teeth; the wheel on the next arbor, that carries the ſecond Satellite, 33 teeth; the next bigger wheel, on the arbor that carries the third Satellite, 43 teeth; and the largeſt wheel of all,

on the arbor that carries the fourth (or outermoſt Satellite), 67: the biggeſt wheel being the uppermoſt, and the ſmalleſt the lowermoſt.

Theſe four wheels muſt be turned by other four, all fixt on a ſolid axis, in an inverted conical manner, with reſpect to the former wheels on the hollow arbors; and then, all the four on the ſolid axis will be turned round in one and the ſame time.

The ſmalleſt wheel (or uppermoſt one) on this axis, muſt have 28 teeth; and turn the wheel of 67 teeth, which carries the fourth Satellite.

The next wheel on the axis muſt have 42 teeth; and turn the wheel of 43 teeth, which carries the third Satellite.

The next bigger wheel below, on the axis, muſt have 65 teeth; and turn the wheel of 33 teeth, which carries the ſecond Satellite. And,

The lowermoſt, and biggeſt wheel on the axis, muſt have 87 teeth; and turn

turn the wheel of 22, which carries the firſt Satellite. Then,

If the clock turns the ſolid axis with all its wheeels round in 7 days, the firſt Satellite will be carried round Jupiter in 1 day, 18 hours, 28 minutes, 57 ſeconds; the ſecond Satellite, in 3 days, 13 hours, 17 minutes, 46 ſeconds; the third, in 7 days, 3 hours, 59 minutes, 54 ſeconds; and the fourth Satellite in 16 days, 18 hours, 0 minutes, 0 ſeconds; which agrees ſo nearly with their revolutions in the heavens, as not to differ ſenſibly, in a long time, from them.

And then, if a piece of black wood be turned, a little conical in its ſhape, having its thickeſt end as broad as the diameter of Jupiter is long, and be made hollow to fix on the back of Jupiter, and have notches cut in it for the Satellites to paſs through: it will repreſent Jupiter's ſhadow; and when the Satellites are in the notches, it will ſhew them to be eclipſed.

The times of the immerſions of the Satellites of Jupiter into his ſhadow,

or of their emerſions from it, may be had from *White's* Ephemeris every year; and if the Satellites are once put juſt entering into the notches for the immerſions, or juſt leaving it for the emerſions, at the proper times by the clock; they will keep right to the times thereof for more than a year afterward, without needing any new adjuſtment. And, in order that they may be ſo ſet, without affecting the wheels that move them, their wires ſhould be fixed into round collars, which go moderately tight on the tops of the four hollow arbors, ſo as they may be carried about Jupiter by the tightneſs of the collars; and yet at any time may be moved, and ſet right by hand.

All the numbers of teeth in the wheels are here copied from Mr. *Romer's* Satellite inſtrument, except thoſe for the ſecond Satellite; where Mr. *Romer* has a wheel of 63 teeth, turning a wheel of 32: inſtead of which, I make a wheel of 65 turn a wheel of 33, which comes much nearer the truth.

About

About 16 years ago, I made one of theſe inſtruments, to be turned with a winch by hand. It had a Dial-plate divided into the months and days of the year, within which was a circle divided into twice twelve hours. On this Plate there were two indexes, one of which was moved round, over all the 365 days of the annual circle, in 365 turns of the winch: and the other index was moved round, over all the 24 hours, in one turn of the winch; by which means I could, in a very ſhort time, ſhew at what times of the days the Satellites would be eclipſed, throughout the whole year. And, after having the above numbers for the motions of the Satellites, any Clock-maker may eaſily conſtruct a machine of this ſort; by which, the times of the Immerſions or Emerſions of the Satellites may be known before-hand, in order to be prepared for obſerving them in the heavens.

How

How to conſtruct an Orrery for ſhewing the annual revolutions of Mercury, Venus, and the Earth, round the Sun, in their proper periodical times; the Moon's motion round the Earth, and round her own axis, with all her different Phaſes: the motions of the Sun, Venus, and the Earth, round their reſpective axes; the viciſſitudes of Seaſons; the retrogade motion of the nodes of the Moon's Orbit; with the times of all the New and Full Moons, and of all the Solar and Lunar Eclipſes.

Let a wheel of 6.12 inches, having 74 teeth, be fixed on the axis of the handle or winch, and turn a wheel of 1.80 inches diameter, having 32 teeth, which turns a wheel of 73 teeth, whoſe diameter is 6.11 inches; and, on the axis of this laſt wheel let there be one of 32 teeth, of 1.80 inches diameter, turning a wheel of 162 teeth, whoſe diameter is 8.97 inches; and that wheel to turn two wheels of 32 teeth each,

each, and diameter 1.80 inches. One of these last wheels of 32 to have a small wheel of 16 teeth on the top of its axis, turning another of the same number and size, and that one to turn such another, on the top of whose axis (inclining 23½ degrees) is the Earth, which turns round in the same time as the winch; each turn answering to 24 hours. The Earth is covered half over with a black cap, and turns freely round within the cap, whose edge represents the boundary of light and darkness, and shews the times of the [apparent] rising and setting of the Sun, as the different places of the Earth emerge from below it, or go in under it.

The other wheel of 32 teeth (turned by the foresaid wheel of 160) has an index on its axis, which goes round a Dial-plate of 24 hours, in the time the Earth turns round its axis. The same wheel of 32 turns one of 64 teeth, whose diameter is 3.60 inches, and turns a wheel of 30 teeth, 1.6 inches

 diameter;

diameter; and on the axis of this wheel is a ſingle threaded ſcrew, turning a wheel of 63 teeth, whoſe diameter is 3 inches, and turns a wheel of 24 teeth, 1.23 inches diameter, which turns a wheel of 63 teeth, 3 inches diameter; which laſt wheel carries the Moon round her orbit in 27 days, 7 hours, 43 minutes, and from change to change, in 29 days, 12 hours, 45 minutes. The firſt wheel of 63 teeth has an index on the top of its axis, which goes round a circle divided into 29½ equal parts in the time of a Lunation; and ſhews the Moon's age every day.

A ſmall wheel of 20 teeth is fixed on a ſocket, among the other work, below the Earth; and by the bar that carries the Moon, a wheel hanging on the bar, of 20 teeth, turns another of the ſame number and ſize, on whoſe hollow axis is the Moon's black cap, which always faces the Sun, and ſhews the Moon's phaſes, as ſhe turns round her axis, which is within the hollow axis of her cap.

On the axis of the winch is a pinion of 8 leaves, turning a wheel of 25 teeth, which turns another of the ſame number, on whoſe axis is a pinion of 7 leaves, $\frac{62}{100}$ parts of an inch in diameter, which turns a wheel of 69 teeth, whoſe diameter is 4.12 inches, and has a pinion of 7 leaves on its axis, turning a wheel of 83 teeth, which is fixed to a frame that contains ſeveral of the above-mentioned wheels within it, and carries the Earth round the Sun in 365 days, 5 hours, 48 minutes, 57 ſeconds. The diameter of the wheel of 83 teeth is 6.12 inches.

On the axis of the laſt mentioned wheel of 69 teeth, is a pinion of 10 leaves, turning a wheel of 73 teeth, whoſe diameter is 5.82 inches, and is fixed to a frame in which are ſeveral other wheels (to be deſcribed by and by) and carries Venus round the Sun in 225 days, 17 hours.

On the axis of the foreſaid wheel of 69 teeth is a wheel of 78, whoſe diameter is 3.68 inches, and turns a wheel

 of

of 64 teeth, whoſe diameter is 2.3 inches, and on the top of whoſe axis the Sun is placed; the axis inclining $7\frac{1}{2}$ degrees, and the Sun turning round by it in 25 days, 6 hours.

In the center of the machine, below the Sun, there are three wheels fixed on the ſtem, round which the whole work moves; the ſtem itſelf being fixed into the bottom of the box which contains the work. The lowermoſt of theſe three wheels is 2.95 inches in diameter, and contains 50 teeth, which take into the teeth of another wheel, of the ſame number and ſize, and this laſt wheel takes into the teeth of another of the ſame number and ſize, for keeping the parallelifm of the Earth's axis in its whole courſe round the Sun; on which parallelifm, the whole variety of the Seaſons depend.

On the axis of the middlemoſt of theſe three wheels of 50 teeth is a wheel of 59, (a little bigger than the wheel of 50) which takes into a wheel of 56 teeth (of a ſomewhat ſmaller ſize) and

and this wheel of 56 moves the Nodes of the Moon's orbit backward, through all the ſigns and degrees of the Ecliptic in $18\frac{2}{3}$ years.

Above the fixed wheel on the middle ſtem, of 50 teeth, is a fixed wheel of 74, whoſe diameter is 6.12 inches, and takes into a pinion of 8 leaves, on the top of whoſe axis is a ſmall wheel of 16 teeth, turning another of the ſame number and ſize, and that turning another of the ſame number and ſize alſo, on whoſe axis, inclining 75 degrees, Venus is turned round in 24 days, 8 hours; which is her diurnal period, according to *Bianchini's* obſervations.

Above the ſaid wheel of 74 teeth, and fixed on the ſame ſtem, is a wheel of 28, whoſe diameter is 1.74 inches; this wheel takes into the teeth of another of the ſame number and ſize, which takes into a third of the ſame number and ſize alſo; and this third wheel keeps the paralleliſm of Venus's axis throughout her whole annual period round the Sun.

On

On the axis of the middlemoſt of theſe three wheels of 28 teeth is another of the ſame number and ſize, which turns a wheel of 18 teeth, whoſe diameter is 1.12 inches, and which turns another of the ſame number and ſize, which carries Mercury round the Sun in 87 days, 23 hours.

Any perſon who is not accuſtomed to the making of Orreries may perhaps be apt to think, that all the abovementioned motions might be performed by fewer wheels; and an expert Clock-maker, by computing the periodical times of the planets revolutions from the numbers of teeth in theſe wheels, might pronounce them to be very inaccurate. But it ought to be conſidered, that there is a very great difference between the rotations of wheels which always keep in the ſame places, and of thoſe which do not only turn round, but are alſo carried round others, continually changing their places and poſitions. As I wanted an Orrery more exact in the annual periods of the planets,

nets, and motion of the Moon round her orbit, than any one I have yet ſeen; the common Orreries being more adapted for reading public lectures upon; where it is ſufficient to ſhew and explain the general phenomena; the makers generally content themſelves with having ſuch numbers as will carry the Earth round the Sun in 365 of its diurnal rotations, the Moon round from change to change in 29½ days, and her nodes round the Ecliptic in 19 years; I have taken the pains to calculate the abovementioned numbers, which are far more exact; and got a good workman to make an Orrery under my inſpection, in which the diameters of the wheels, and their numbers of teeth are exactly deſcribed; and which I now give freely to thoſe who chooſe to work by them. The man who made this Orrery had never made any thing of the kind before, and he is now dead. An ingenious and worrthy lady has it now in her poſſeſſion, whoſe father was one of my firſt and beſt friends in London.

Another Orrery.

About twelve years ago, I made a large wooden Orrery, for ſhewing only the motions of the Earth and Moon, with the retrograde motions of her nodes, and the phenomena ariſing from all their motions. The Earth had not its diurnal and annual motions carried on by means of a winch, but by hand; and as the Earth was moved round the Sun, the Moon was carried round the Earth in her orbit, and her nodes had their retrograde motions. As there is ſomething very particular and ſimple in the conſtruction of this machine; and as the Moon's motion in it will not vary above one degree from the truth in 304 years; and as it anſwers as well in Leap years as in common years; and has only ſeven wheels and one pinion in it; I ſhall here mention its uſe, but muſt beg to be excuſed from deſcribing the poſition of its wheels, and their numbers of teeth, because I intend to inſtruct my ſon,

ſon, if both he and I live till the proper time, how to make it for his own benefit. Beſides, it would be very difficult to make it intelligible by a deſcription, without ſeeing it; eſpecially as ſome of the wheels are not only divided into very uncommon numbers of teeth, but alſo that, in ſome of the wheels, equal numbers of teeth are contained in unequal ſpaces; for ſhewing the inequality of the Earth's annual motion round the Sun, and of the Moon's motion round her orbit. It ſhews the following matters very readily.

The lengths of days and nights at all places of the Earth, and at all times of the year; with all the viciſſitudes of Seaſons. The Sun's place in the Ecliptic on any given day of the year, and time of the day; with his Declination, Altitude, and Azimuth at any time; alſo his Amplitude, and the time of his riſing and ſetting. The time of the day, by the obſerved Altitude or Azimuth of the Sun. The variation of the compaſs, in any place, whoſe latitude is

known, by a ſingle obſervation of the Sun's Altitude, taken at any time, either in the Forenoon or Afternoon. The Moon's periodical and ſynodical revolution, with her rotation on her axis, and different phaſes. The retrogade motion of her Nodes, and direct motion of her Apogee. Her mean Anomaly and elliptic Equation, by which her true place in her Orbit is very nearly found at any time. Her Latitude, Declination, Altitude, and Azimuth, at any time when ſhe is above the horizon. Her Amplitude, and the time of her riſing and ſetting, however affected by her Latitude. The times of all the New and Full Moons, and of all the Solar and Lunar Eclipſes, within the limits of 6000 years before or after the Chriſtian Æra; with an eaſy method of rectifying the Machine, in leſs than two minutes of time, for the beginning of any given year within theſe limits: and when it is once rectified, it will keep right for 304 years either backwards or forwards; at the end of which

time,

time, the Moon muſt be ſet one degree forward in her orbit. The ſmall difference in the time of the Moon's riſing, in Harveſt, throughout the week in which ſhe is Full; and the great difference in the time of her ſetting during that week. The Receſſion of the Equinoctial points, in the Ecliptic. The Phenomena of the Tides, and the cauſes of many apparent irregularities in their heights, and times of ebbing and flowing.

The Mechanical Paradox.

This is a ſmall kind of Orrery, which I contrived and made about fifteen years ago. It has only five wheels, and ſhews the Seaſons, the retrograde motion of the Moon's Nodes, and the mean times of Eclipſes of the Sun and Moon. I gave it the above name, becauſe there is one wheel in it as thick as three of the others; and *that* wheel takes fairly and equally deep into the teeth of theſe three other wheels (which are quite indepen-

 dent

dent of, and unconnected with, each other); and yet, the thick wheel affects the three wheels in such a manner, and at the same time, as to turn the uppermost of them forward, the middlemost backward, and the lowermost no way at all. For a Copper-plate of this machine, with a printed description in which the paradox is solved, I refer the reader to a Shilling Pamphlet, sold at Mr. Cadell's Shop, opposite Catherine Street in the Strand.

A short account of the Silk Mills at Derby.

In these Mills are 26586 wheels, and 97746 movements, continually working except on Sundays. This grand machine is disposed in four stories of large rooms above one another; and the whole is actuated by one great Water-wheel, which goes round three times in a minute. In each time of its going round, 73728 yards of Silk are twisted: so that, in 24 hours, 318504960 yards are

are twiſted. The Water-wheel is kept conſtantly going; but on Sundays it is diſengaged from all the reſt of the work. Any part of theſe movements may be ſtopt without the leaſt prejudice or interruption to the reſt.

Wondrous Machine! Thy curious Fabric ſhews
How far the power of human wiſdom goes!
Where many thouſand movements all attend
Upon a WHEEL, and on THAT Cauſe depend.
Sceptic, advance! propoſe *thy* ſcheme of wit,
That faith to reaſon always muſt ſubmit.
Whence learn'd theſe movements to obey command?
Who taught them how to roll, and when to ſtand?
Was it by chance this curious fabric came?
Or did ſome thought precede, and rule the Frame?
Worthy the Mortal, on whoſe Soul, confeſt,
His GREAT CREATOR's Image ſtands impreſt!
Now turn from Earth to Heaven thy doubting eyes,
And read th' amazing Glories of the Skies!
Worlds without number roll in different Spheres,
Keep to their Seaſons, and complete their years.
Five thouſand circuits, made with equal force,
The Earth has finiſh'd by its annnal Courſe.
The Sun diſpenſes beams of genial Light,
And lends his rays to cheer the gloomy night.
STUPENDOUS POWER and THOUGHT! Enquire no more:
Own the FIRST MOVER; and, convinc'd, ADORE.

Rules

Rules for finding the correſponding years of the Julian Period with the years of the world, and years before and ſince the birth of CHRIST; *ſuppoſing (with Mr.* Bedford, *in his Scripture Chronology) that the Creation of the World was in the 706th year of the Julian Period; and that the birth of* CHRIST *was (according to the vulgar Æra thereof) in the 4713th year of the Julian Period.*

From any given year of the Julian period ſubtract 706, and the remainder will be the years of the world's age.

If the number of the given year of the Julian period be leſs than 4713, ſubtract it from 4713; and the remainder will be the number of years before the year of Chriſt's birth.

If the given year of the Julian period is greater than 3967, ſubtract 3967 from it; and the remainder will be the number of years after the famous Æra of *Nabonaſſar.*

Subtract

Subtract 1 from any given year of the Julian period, and divide the remainder by 4; if nothing remains, the given year is a Leap year: but if 1, 2, or 3 remains, it is the firſt, ſecond, or third year after Leap-year, in the Old Stile.

If any year before the year of Chriſt's birth be given, ſubtract its number from 4713, and the remainder will be the year of the Julian period. And if you ſubtract the ſaid given year from 4007, the remainder will be the years of the world's age.

If any year after the year of Chriſt's birth be given, add 4713 to it, and the ſum will be the year of the Julian period; or if you add 4007 to it, the ſum will be the years of the world's age.

If any year of the world's age is given, add 706 to it, and the ſum will be the year of the Julian period. If the given year of the world be leſs than 4007, ſubtract it from 4007; and the remainder will be the number

of

of years before the year of Chriſt's birth. But, if the given year of the world be more than 4007, ſubtract 4007 from it; and the remainder will be the number of years after the year of Chriſt's birth.

A Table of remarkable Æras and Events.

	Julian Period.	World's Age.	Before Chriſt.
1. The Creation of the World	706	0	4007
2. The Flood — — —	2362	1656	2351
3. The *Aſſyrian* monarchy founded by *Nimrod* — — —	2537	1831	2176
4. The birth of *Abraham* —	2714	2008	1999
5. The deſtruction of *Sodom* and *Gomorrah* — — —	2816	2110	1897
6. The kingdom of *Athens* founded by *Cecrops* — —	3157	2451	1556
7. *Moſes* receives the ten Commandments from God —	3222	2516	1491
8. The *Iſraelites* enter *Canaan*	3262	2556	1451
9. The deſtruction of *Troy* —	3529	2823	1184
10. The beginning of king *David*'s reign — —	3650	2944	1063
11. The founding of *Solomon*'s Temple — — —	3701	2995	1012
12. The *Argonautic* Expedition	3776	3070	937
13. *Lycurgus* formed his excellent Laws — —	3829	3103	884
14. *Arbaces*, firſt king of the *Medes*	3838	3132	875
15. *Mandaucus*, the ſecond	3865	3159	848
16. *Soſarmus*, the third —	3915	3029	798
17. The beginning of the *Greek Olympiads* — —	3938	3232	775
18. *Artica*, the fourth king of the *Medes* — —	3945	3239	768

19. The

	Julian Period.	World's Age.	Before Christ.
19. The *Catonian Epocha* of the building of *Rome*. — —	3961	3255	752
20. The *Æra* of *Nabonassar*	3967	3261	746
21. The destruction of *Samaria* by *Salmaneser* — —	3992	3286	721
22. The first Eclipse of the Moon on record — —	3993	3287	720
23. *Cardicea*, the fifth king of the *Medes* — —	3996	3290	717
24. *Phraortes*, the sixth —	4058	3352	655
25. *Cyaxares*, the seventh —	4080	3374	633
26. The first *Babloynish* captivity by *Nebuchadnezzar* —	4107	3401	606
27. The long war ended between the *Medes* and *Lydians* —	4111	3405	602
28. The second *Babylonish* captivity, and birth of *Cyrus* —	4114	3408	599
29. The destruction of *Solomon*'s Temple — —	4125	3419	588
30. *Nebuchadnezzar* struck with madness. — — —	4144	3438	569
31. *Daniel's* vision of the four monarchies — —	4158	3452	555
32. *Cyrus* begins to reign —	4177	3471	536
33. The battle of *Marathon* —	4223	3517	490
34. *Artaxerxes Longimanus* begins to reign — — —	4249	3543	464
35. The beginning of *Daniel*'s seventy weeks of years —	4256	3350	457
36. The beginning of the *Peloponnesian* war — —	4282	3576	431
37. *Alexander*'s victory at *Arbela*	4383	3677	330
38. His death — —	4390	3684	323
39. The captivity of 100000 Jews by king *Ptolemy* — —	4393	3687	320
40. The *Colossus* of *Rhodes* thrown down by an earthquake	4491	3875	222
41. *Antiochus* defeated by *Ptolemy Philopater* — —	4496	3700	217
42. The famous ARCHIMEDES murdered at *Syracuse* —	4506	3800	207
43. *Jason* butchered the inhabitants of *Jerusalem* — —	4543	3837	170
44. *Corinth* taken and plundered by Consul *Mummius* —	4567	3861	146

	Julian Period.	World's Age.	Before Christ.
45. *Julius Caesar* invades *Britain*	4659	3953	54
46. He corrects the calendar	4667	3961	46
47. Is killed in the Senate-house	4671	3965	42
48. *Herod* made king of *Judea*	4673	3967	40
49. The battle at *Actium* —	4683	3977	30
50. *Agrippa* builds the *Pantheon* at *Rome* — —	4668	3982	25
51. The true *Æra* of CHRIST's birth — — —	4709	4003	4
52. The death of *Herod* —	4710	4004	3
			Since Christ.
53. The *Dionysian*, or vulgar *Æra* of CHRIST's birth —	4713	4007	0
54. The true year of his Crucifixion — — —	4746	4040	33
55. The destruction of *Jerusalem*	4783	4077	70
56. *Adrian* built the long wall in *Britain* — — —	4833	4127	120
57. *Constantius* defeated the *Picts* in *Britain* — —	5019	4313	306
58. The council of *Nice* —	5038	4332	725
59. The death of *Constantine* the Great — —	5050	4344	337
60. The *Saxons* invited into *Britain*	5158	4452	445
61. The *Arabian Hegira*, or flight of *Mohammed* — —	5335	4629	662
62. The death of *Mohammed* —	5343	4637	630
63. The *Persian Yesdegird* —	4344	4638	631
64. The art of Printing discovered	6153	5447	1440
65. The Reformation begun by *Martin Luther* —	6230	5524	1517
66. *Oliver Cromwell* died —	6371	5665	1658
67. Sir ISAAC NEWTON born at *Woolstrope* in *Lincolnshire*, December 25 — — —	6355	5649	1642
—— went to *Trinity College* in *Cambridge* — —	6373	5667	1660
—— was elected Fellow of that College — — —	6380	5674	1667
—— invented the Fluxions —	6382	5676	1669
—— made Professor of Mathematics, in the room of Dr. *Barrow*	6382	5676	1669
—— published his *Principia* —	6400	5684	1687
—— exerted himself for Religion	6401	5685	1688

—— made

	Julian Period.	World's Age.	Since Christ.
—— made Prefident of the Royal Society — —	6416	5700	1703
—— knighted by Queen ANNE —	4618	5702	1705
—— died, March 20 —	4640	5734	1727

In this Table, the years both before and fince CHRIST are reckoned exclufive from the year of his birth.

The year of our SAVIOUR'S *Crucifixion aſcertained; and the darkneſs at the time of his Crucifixion proved to be ſupernatural.*

Concerning the time of our Saviour's entering upon his public miniſtry (which may be called the time of his appearance, becauſe, till then, he was not publicly known, ſo as to be talked of), and alſo concerning the time of his death, there is a very remarkable propheſy in the IXth chapter of the book of *Daniel*, from the 24th verſe to the end; which is in our Engliſh Tranſlation as follows.

Ver. 24. *Seventy weeks are determined upon thy people, and upon thy holy city, to finiſh the tranſgreſſion, and to make an end of ſins, and to make reconciliation for iniquity, and to bring in everlaſting righteouſneſs, and to ſeal up the viſion and propheſy, and to anoint the moſt holy.*

25. *Know therefore and underſtand, that from the going forth of the command-*

commandment to reſtore and build Jeruſalem, unto the Meſſiah the prince, ſhall be ſeven weeks; and threeſcore and two weeks the ſtreet ſhall be built again, and the wall, even in troublous times.

26. *And after threeſcore and two weeks ſhall Meſſiah be cut off, but not for himſelf: and the people of the prince that ſhall come, ſhall deſtroy the city and the ſanctuary, and the end thereof ſhall be with a flood, and unto the end of the war deſolations are determined.*

27. *And he ſhall confirm the covenant with many for one week: and in the midſt of the week he ſhall cauſe the ſacrifices and oblations to ceaſe, and for the overſpreading of abomination he ſhall make it deſolate, even until the conſummation, and that determined ſhall be poured upon the deſolate.*

In the Hebrew, there are no ſtops nor pointings to any words or ſentences; and in the above tranſlation, one part of the 25th verſe is moſt injudiciouſly pointed with a ſemi-colon at *ſeven weeks;*

weeks; which ought to run thus, *ſeven weeks and threeſcore and two weeks.*

In the 24th verſe, what we have rendered *propheſy*, is *prophet* in the original: and in ſome tranſlations, which I have procured from thoſe who underſtand the Hebrew very well, inſtead of *viſion and propheſy*, it is rendered *viſions and prophets.*

In ver. 27. where we have it *the midſt of the week*, all the tranſlations I have procured render it the *half part of the week*; which may be taken either for the firſt or laſt half part of it.

In the ſame verſe, where we have it *And he ſhall confirm the covenant with many for one week*; ſome tranſlations render it *And in one week a covenant ſhall be confirmed with many.* Now let the whole be put together agreeable to this tranſlation, without dividing it into different verſes (which is only of modern invention) but pointing it here and there for the ſake of reading; and it will run thus.

Seventy

Seventy weeks are determined upon thy people and thy holy city to finish the transgressions and to make an end of sins; and to make reconciliation for iniquity, and to bring in everlasting righteousness, and to seal up the visions and prophets, and to anoint the most holy. Know therefore, and understand, that from the going forth of the commandment to restore and build Jerusalem, unto the Messiah the prince, shall be seven weeks and threscore and two weeks: the street shall be built again, and the wall even in† troublous times. And after threescore and two weeks shall Messiah be cut off, but not for himself. (And the people of the prince that shall come shall destroy the city and sanctuary, and the end thereof shall be with a flood; and unto the end of the war desolations are determined). And in one week a covenant shall be confirmed with many, and in half part of the week* HE ‡ *shall abolish the sacrifices and offerings.*

* Some translate this, *the holy of holies*, and Mr. Purver, *the very holy one.*

† By most translators, *in the straitness of times.*

‡ The Messiah.

ings. And for the overſpreading * *of abominations he ſhall make deſolate even unto conſuming; and that which is determinedſhall be poured upon the deſolate.*

'Tis evident, that the firſt part of this propheſy relates to the coming of Chriſt; to his being put to death, *not for himſelf*, but for the ſins of mankind, by which great ſacrifice he was to put an end to all other ſacrifices and offerings; to his introducing the righteouſneſs of ages, and ſealing up (or putting an end to) propheſies. And that the latter part mentions the deſtruction of Jeruſalem, in a very emphatic and ſtriking manner.

In the ſeventh chapter of *Ezra*, we have an account of a very ample and full commiſſiion (or commandment) which was given by king *Artaxerxes* (who was called *Artaxerxes Longimanus*) to *Ezra*, to go up to Jeruſalem, in order to repair that city, and reſtore the ſtate of the Jews; and that *Ezra* took his

* *Wing* in the Hebrew.

journey

journey on the firſt day of the firſt month, *viz.* the month *Niſan*; which began about the time of the vernal equinox. And on the 14th day of that month (reckoned from the New Moon, at which the month began) the paſſover was always kept; for *Joſephus* * expreſſly ſays, "The paſſover was kept on the "14th day of the month *Niſan*, according "to the Moon, when the Sun was in "Aries." And the Sun always enters the ſign Aries at the time of the vernal equinox.

This commandment was given in the 7th year of *Artaxerxes*'s reign, and *that* year (according to *Ptolemy*'s canon, the rectitude of which was ſcarce ever called in queſtion) was the 4256th year of the Julian period; and from the vernal equinox in that year, we are to count the above-mentioned ſeventy weeks to the death of Chriſt. For, as the accompliſhment of the prophesy muſt end with the expiation of ſin, we cannot ſuppoſe theſe weeks to end at any other time.

* Lib. i. cap. 10.

But, if we count many revolutions of 70 common weeks, from the time of the Jewiſh paſſover in the year of the Julian period 4256, we ſhall find that no Meſſiah or Saviour did appear on the Earth within that ſpace of time: nor will theſe reckonings lead us from one Paſſover to another. And it is certain, from the four Goſpels, that Chriſt was crucified at the time of the Paſſover; and St. *John*, chap. xviii. ver. 28. is ſo particular, as to inform us that our Saviour was crucified on the very day that the Paſſover was to be eaten by the Jews, who would not defile themſelves by mixing with the multitude early in the morning, at the time of his trial. —— From theſe circumſtances it is plain that theſe prophetic weeks mean ſomething very different from the weeks by which we commonly reckon.

In the Old Teſtament, we read of weeks of years, as well as weeks of days. For, as every ſeventh day was to be a ſabbath for man, on which he was to reſt from his labour; ſo every ſeventh year

year was to be a ſabbath for the land, in which it was to reſt from tillage. Let us therefore take theſe 70 weeks to be weeks of years, making 490 years in all; and the reckoning will lead us from the Paſſover in the year of the Julian period 4256, to the Paſſover in the year 4746, which was the 33d year of our Saviour's age, accounted from the vulgar æra of his birth.

It is expreſsly foretold in this propheſy, that from the time of the commandment's being given out to reſtore and build Jeruſalem, to the Meſſiah the prince, (or to the time of his appearing in his public character) there would be ſeven weeks and threeſcore and two weeks; or 69 weeks in all: the firſt ſeven of which, being the ſtraiteſt or ſhorteſt of the times, conſiſting of 49 years, we may very well allot to the repairing of Jeruſalem; after which, there ſhould be threeſcore and two weeks, or 434 years, to the public appearance of the Meſſiah: and then there remained only one week, or ſeven

years, for the public miniſtry; which, I apprehend, is meant by *confirming the covenant with many.*

But as ſome of the Tranſlations which I have procured, ſay, concerning that week, *And in one week a covenant ſhall be confirmed with many*; and all of them have it, *and in half part of the week* (which might be either the firſt or laſt half of it) HE *ſhall aboliſh the ſacrifices and offerings*; it does not appear that the Meſſiah is brought in for the whole of the ſeventieth week, but only for one half of it, *in confirming* (or eſtabliſhing) *the* new and everlaſting *covenant* of the Goſpel; by which, *the righteouſneſs of ages*, mentioned in the firſt verſe of the propheſy, ſeems to be plainly meant.

And when we conſider, that CHRIST's meſſenger, *John* the Baptiſt, preached ſo long before Chriſt took the public miniſtry upon himſelf, as that he acquired great fame in many countries around, which could not be done in a ſhort time, we may believe that the laſt verſe of the propheſy allots the firſt half

half of the seventieth week (or three years and an half) to the time of *John*'s preaching; at the end of which time he baptized Christ, who was then entering into the thirtieth year of his age (according to St. *Luke*) and then Christ took his public ministry upon himself for the remaining half of the seventieth week; at the end of which he was cut off by the wicked and self-hardened Jews, and so put a *virtual* end to all their sacrifices and offerings; which *finally* ended with the destruction of their city and temple about 37 years after.

So that, in the first place, taking the whole of the prophesy together, as in *ver.* 25, and then dividing it into four different periods or parts as above mentioned; it will very naturally run thus,

	Weeks.		Years.
From the time of *Ezra's* receiving the commandment to repair *Jerusalem*, until the expiation of Sin by CHRIST —	70	or	490
For the time of these repairs ———	7	or	49
From the finishing of these repairs to the coming of CHRIST by his messenger *John* the Baptist —— ——	62	or	434
From that time to the end of *John's* ministry, and the baptism of CHRIST —	$\frac{1}{2}$	or	$3\frac{1}{2}$
From thence to the end of CHRIST's ministry, by his death on the Cross ——	$\frac{1}{2}$	or	$3\frac{1}{2}$
In all —	70	or	490

For a very full illuſtration of this matter, I refer the reader to Dr. *Prideaux*'s Connexion of the hiſtories of the Old and New Teſtament.

The beginning of theſe ſeventy weeks of years being found to be the year of the Julian period 4256, at the time of the Jewiſh Paſſover, their ending muſt have been at the Paſſover in the year of the Julian period 4746, in the 33d year after the year of Chriſt's birth: and conſequently, according to this propheſy, our Saviour was crucified in the 4746th year of the Julian period.

'Tis plain from all the four Goſpels, that the Crucifixion was on a Friday; becauſe it was on the day next before the Jewiſh Sabbath; and, as above mentioned, on the day the Paſſover was to be eaten (at leaſt) by many of the Jews.

The Jewiſh year conſiſted of twelve months, as meaſured by the Moon, which contains 354 days; to which they either added 11 days every year, in order to make their years keep pace with the Sun's courſe of 365 days; or 30 days in three

three years. So that, although their months were Lunar, their years were Solar. And they always celebrated the Paſſover on the fourteenth day of the firſt Lunar month, reckoning from the firſt time of their ſeeing the New Moon; which, eſpecially at that time of the year, might be when ſhe was about 24 hours old: and conſequently their fourteenth day of the month fell upon the day of Full Moon; and, according to *Joſephus*, they always kept the Paſſover at the time of the Full Moon next after the vernal equinox.

But the Full Moon day on which our Saviour was crucified fell on Friday. And as 12 Lunar months want 11 days of 12 Solar months, the Paſſover Full Moons (as well as all others) fall 11 days back every year; which being more than a week, by four days, makes it, that, in a few neighbouring years, there cannot be two Paſſover Full Moons on the ſame day of the week. And when this anticipation would have made the Paſſover Full Moon fall before the equinoctial

noctial day, they set it a whole month forward, to have it at the first Full Moon after the vernal Equinox; which puts it off the same day of the week again.

The dispute among chronologers, about the year of our Saviour's crucifixion, is limited within four or five years at most. And it certainly was in the year in which the Passover Full Moon fell on a Friday.

And I find, by calculation, that the only Passover Full Moon which fell on a Friday, from the 20th year after our Saviour's birth to the 40th, was in the 4746th year of the Julian period; which was the 33d year of his age, reckoning from the beginning of the year next after that of his birth, according to the vulgar *Æra* thereof: and the said Passover Full Moon was on the third day of April.

And thus we have an astronomical demonstration of the truth of this ancient prophesy, seeing that the prophetic year of the Messiah's being cut off

was the very ſame with the aſtronomical.

Beſides, we have the teſtimony of a heathen author, which agrees with the ſame year. For *Phlegon* informs us, that in the fourth year of the 202d *Olympiad* (which was the 4746th year of the Julian period, and the 33d year after the year of Chriſt's birth) there was the greateſt eclipſe of the Sun that ever was known; for the darkneſs laſted three hours in the middle of the day: which could be no other than the darkneſs on the Crucifixion-day; as the Sun never was totally hid above four minutes of time, from any part of the Earth, by the interpoſition of the Moon.

If *Phlegon* had been an aſtronomer, he would have known that the ſaid darkneſs could not have been occaſioned by any regular eclipſe of the Sun; as the Moon was then in the oppoſite ſide of the heavens, on account of her being Full. And as there is no other body than the Moon that ever comes be-

tween the Sun and the Earth, it is evident that the darkneſs at the crucifixion was miraculous, being quite out of the ordinary courſe of nature.

There have been great difficulties about our Saviour's eating the Paſchal lamb on the evening of the day before it was eaten by the Jews. But I apprehend this difficulty may be eaſily removed, when we conſider that the Jews began their day in the evening, and ended it in the next following evening. So that, although it was on a different day, according to our way of reckoning, it was ſtill the ſame day according to theirs. And we do not find that they brought in his eating the lamb on the Thurſday evening as any accuſation againſt him: which they would undoubtedly have been glad to do, if they could have made a handle of it for that purpoſe.

A Table

A Table shewing the Dominical Letters for years before the year of Christ's birth.

Old Stile.				Hundreds of Years.						
Years less than an Hundred.				0 700 1400 2100 2800 3500 4200 4900	100 800 1500 2200 2900 3600 4300 5000	200 900 1600 2300 3000 3700 4400 5100	300 1000 1700 2400 3100 3800 4500 5200	400 1100 1800 2500 3200 3900 4600 5300	500 1200 1900 2600 3300 4000 4700 5400	600 1300 2000 2700 3400 4100 4800 5500
0	28	56	84	D C	C B	B A	A G	G F	F E	E D
1	29	57	85	E	D	C	B	A	G	F
2	30	58	86	F	E	D	C	B	A	G
3	31	59	87	G	F	E	D	C	B	A
4	32	60	88	B A	A G	G F	F E	E D	D C	C B
5	33	61	89	C	B	A	G	F	E	D
6	34	62	90	D	C	B	A	G	F	E
7	35	63	91	E	D	C	B	A	G	F
8	36	64	92	G F	F E	E D	D C	C B	B A	A G
9	37	65	93	A	G	F	E	D	C	B
10	38	66	94	B	A	G	F	E	D	C
11	39	67	95	C	B	A	G	F	E	D
12	40	68	96	E D	D C	C B	B A	A G	G F	F E
13	41	69	97	F	E	D	C	B	A	G
14	42	70	98	G	F	E	D	C	B	A
15	43	71	99	A	G	F	E	D	C	B
16	44	72		C B	B A	A G	G F	F E	E D	D C
17	45	73		D	C	B	A	G	F	E
18	46	74		E	D	C	B	A	G	F
19	47	75		F	E	D	C	B	A	G
20	48	76		A G	G F	F E	E D	D C	C B	B A
21	49	77		B	A	G	F	E	D	C
22	50	78		C	B	A	G	F	E	D
23	51	79		D	C	B	A	G	F	E
24	52	80		F E	E D	D C	C B	B A	A G	G F
25	53	81		G	F	E	D	C	B	A
26	54	82		A	G	F	E	D	C	B
27	55	83		B	A	G	F	E	D	C

A Table shewing the Dominical Letters for years after the year of Christ's birth.

Old Stile.				Hundreds of Years.						
Years less than an Hundred.				0 700 1400 2100 2800 3500 4200 4900	100 800 1500 2200 2900 3600 4300 5000	200 900 1600 2300 3000 3700 4400 5100	300 1000 1700 2400 3100 3800 4500 5200	400 1100 1800 2500 3200 3900 4600 5300	500 1200 1900 2600 3300 4000 4700 5400	600 1300 2000 2700 3400 4100 4800 5500
0	28	56	84	D C	E D	F E	G F	A G	B A	C B
1	29	57	85	B	C	D	E	F	G	A
2	30	58	86	A	B	C	D	E	F	G
3	31	59	87	G	A	B	C	D	E	F
4	32	60	88	F E	G F	A G	B A	C B	D C	E D
5	33	61	89	D	E	F	G	A	B	C
6	34	62	90	C	D	E	F	G	A	B
7	35	63	91	B	C	D	E	F	G	A
8	36	64	92	A G	B A	C B	D C	E D	F E	G F
9	37	65	93	F	G	A	B	C	D	E
10	38	66	94	E	F	G	A	B	C	D
11	39	67	95	D	E	F	G	A	B	C
12	40	68	96	C B	D C	E D	F E	G F	A G	B A
13	41	69	97	A	B	C	D	E	F	G
14	42	70	98	G	A	B	C	D	E	F
15	43	71	99	F	G	A	B	C	D	E
16	44	72		E D	F E	G F	A G	B A	C B	D C
17	45	73		C	D	E	F	G	A	B
18	46	74		B	C	D	E	F	G	A
19	47	75		A	B	C	D	E	F	G
20	48	76		G F	A G	B A	C B	D C	E D	F E
21	49	77		E	F	G	A	B	C	D
22	50	78		D	E	F	G	A	B	C
23	51	79		C	D	E	F	G	A	B
24	52	80		B A	C B	D C	E D	F E	G F	A G
25	53	81		G	A	B	C	D	E	F
26	54	82		F	G	A	B	C	D	E
27	55	83		E	F	G	A	B	C	D

Dominical Letters for the New Stile.

1752	B A	1787	G
1753	G	1788	F E
1754	F	1789	D
1755	E	1790	C
1756	D C	1791	B
1757	B	1792	A G
1758	A	1793	F
1759	G	1794	E
1760	F E	1795	D
1761	D	1796	C B
1762	C	1797	A
1763	B	1798	G
1764	A G	1799	F
1765	F	1800	E
1766	E	1801	D
1767	D	1802	C
1768	C B	1803	B
1769	A	1804	A G
1770	G	1805	F
1771	F	1806	E
1772	E D	1807	D
1773	C	1808	C B
1774	B	1809	A
1775	A	1810	G
1776	G F	1811	F
1777	E	1812	E D
1778	D	1813	C
1779	C	1814	B
1780	B A	1815	A
1781	G	1816	G F
1782	F	1817	E
1783	E	1818	D
1784	D C	1819	C
1785	B	1820	B A
1786	A	1821	G

A Table shewing the days of the Months for ever, both in the Old and New Stile, by the Dominical Letters.

Months.	A	B	C	D	E	F	G
Janu. 31 Octob. 31	1	2	3	3	5	6	7
	8	9	10	11	12	13	14
	15	16	17	18	19	20	21
	22	23	24	25	26	27	28
	29	30	31	—	—	—	—
Feb. 28 Mar. 31 Nov. 30	—	—	—	1	2	3	4
	5	6	7	8	9	10	11
	12	13	14	15	16	17	18
	19	20	21	22	23	24	25
	26	27	28	29	30	31	—
April 30 July 31	—	—	—	—	—	—	1
	2	3	4	5	6	7	8
	9	10	11	12	13	14	15
	16	17	18	19	20	21	22
	23	24	25	26	27	28	29
	30	31	—	—	—	—	—
August 31	—	—	1	2	3	4	5
	6	7	8	9	10	11	12
	13	14	15	16	17	18	19
	20	21	22	23	24	25	26
	27	28	29	30	31	—	—
Sept. 30 Dec. 31	—	—	—	—	—	1	2
	3	4	5	6	7	8	9
	10	11	12	13	14	15	16
	17	18	19	20	21	22	23
	24	25	26	27	28	29	30
	31	—	—	—	—	—	—
May 31	—	1	2	3	4	5	6
	7	8	9	10	11	12	13
	14	15	16	17	18	19	20
	21	22	23	24	25	26	27
	28	29	30	31	—	—	—
June 30	—	—	—	—	1	2	3
	4	5	6	7	8	9	10
	11	12	13	14	15	16	17
	18	19	20	21	22	23	24
	25	26	27	28	29	30	

By

By the preceding Tables (p. 195, 196, 197) the day of the month anſwering to any given day of the week, and the day of the week anſwering to any given day of the month, may be found, in the Old Stile, within the limits of 5500 years before the year of Chriſt's birth, and 5500 years after it: and, in the New Stile, from *A. D.* 1752, to 1821 incluſive; as follows.

1. For any given year before Chriſt, look for the compleat hundreds of that year (when its number amounts to hundreds) at the head of the Table on page 195, and for the years below or leſs than an hundred, to make up the number of the given year, at the left hand; and where the columns meet, you have the Dominical letter for the given year. Thus, ſuppoſe the Dominical letter was required for the 585th year before the year of Chriſt 1, which was the 584th before the year of his birth. Under 500 at the head of the Table, and againſt 84 at the left hand, I find FE, which is the Dominical letter required; and

and ſhews the ſaid year to have been a Leap year; as every Leap year has two Dominical letters, the firſt of which ſerves for January and February, and the laſt for all the reſt of the year. The Dominical Letter for any given year after the birth of Chriſt is found in the ſame way by the Table in page 196. Thus, ſuppoſe it was required for the year 1747; I look for 1700 at the head of the Table, and downward thence, in that column againſt 47 at the left hand, I find D; which ſhews that D was the Dominical letter for the year 1747. Theſe two Tables ſhew the Dominical for the Old Stile: and the Table on page 197 ſhews it for the New Stile, from *A. D.* 1752 to *A. D.* 1821.

2. Having found the Dominical letter for the given year, look for that letter at the top of the Table ſhewing the days of the months (pag. 197) and under the ſaid letter, you have all the days of the months which are Sundays in that year, in the diviſions of the months.

Under

Under the next letter toward the right hand, all the days in the column are Mondays; thoſe under the next are Tueſdays; and ſo on. When you are out at the right hand of the Table, go back to the left, and ſo reckon on according to the order of the days of the week.

Thus, ſuppoſe for the 585th year before Chriſt, for which the Dominical letter (or letters) was FE; the firſt ſerving for January and February, and the laſt for all the reſt of the year; in the Table, pag. 197, I find, under F, the 6th, 13th, 20th and 27th of January; and the 3d, 10th, 17th and 24th of February: and then, under E, I find the 2d, 9th, 16th, 23d and 30th of March and November; the 5th, 12th, 19th and 26th of October; the 6th, 13th, 20th and 27th of April and July; the 3d, 10th, 17th, 24th and 31ſt of Auguſt; the 7th, 14th, 21ſt and 28th of September and December; the 4th, 11th, 18th, and 25th of May; and the 1ſt, 8th, 15th, 22d and 29th of

of June; which being all Sundays in that year, the reſt of the days of the months anſwering to given days of the week, are eaſily found. For example; if it was required to know on what day of the week the 28th of May fell, in the abovementioned year; I look for the 28th of May in the Table, and I find A ſtands at the top of the column in which that day is found: and, as the 25th of May fell on Sunday, 'tis plain that the 28th of May muſt have been on Wedneſday.

Again, ſuppoſe it was required to find on what day of the week Chriſtmaſs-day will fall upon, in the year 1767, New Stile. The Dominical letter for that year is D. Then, under D in the diviſion for December, in the Table, I find that the 6th, 13th, 20th and 27th are Sundays; and conſequently, as the 20th of December falls on Sunday, the 25th (or Chriſtmas-day) muſt be on Friday. More examples would be ſuperfluous.

How to divide circles and ſtraight lines, into any given number of equal parts, whether odd or even.

When the given number of equal parts, into which a circle, or a ſtraight line is to be divided, are even, and can be divided by 2, 3, 4, &c. the operation is too eaſy to need any deſcription: but when the given number of parts is odd, as 365, 59, or 31 (which are numbers often wanted) 'tis found to be difficult to divide them, even by a great many trials with the compaſſes.

In order to avoid this difficulty I ſhall ſhew a method, by which it is as eaſy to divide either a given circle, or a given ſtraight line, into any odd number of parts, as into any even number; and have all the ſpaces between the diviſion lines as equal among themſelves as is ſufficient for the purpoſe: provided the operator has a good ſector, knows how to open it till the two 60's on the line of chords are as far aſunder, when tried by

by the compaſſes, as is equal to the length of the radius, or ſemidiameter, of the given circle.

There can be no given number of odd diviſions or parts, but may have as many ſubtracted from it as will reduce it to an even number, which may be biſected, triſected, or quartered, &c. And therefore, by finding the length of an arc in the circle that will bear the ſame proportion to the odd number taken off, as the whole circle bears to the whole given number; this arc may be eaſily divided into as many equal parts as are contained in the odd number which was ſubtracted; and then, the remaining number being even, the remaining part of the circle may be eaſily divided into that number.

All circles contain 360 degrees. Therefore, as the whole number of parts, into which the circle muſt be divided, is to 360, ſo is the number of parts ſubtracted to the number of degrees, and parts of a degree, contained in the arc in which they muſt be di-

 vided.

vided. Thus, ſuppoſe it was required to divide a given circle into 365 equal parts: ſubtract five of theſe parts, and there will remain 360, which may be firſt divided into ſix equal parts, each of theſe again into ſix, and each of theſe laſt into ten; by which there will be 360 in all. Now ſay, as 365 parts is to 360 degrees (the whole circle) ſo is the five parts ſubtracted to the arc they will fill; which arc, by the calculation, will be found to be 4 degrees, and 93 hundred parts of a degree; which is a little more than 9 tenths.

Therefore, having taken the length of the ſemidiameter of the given circle by your compaſſes; open the ſector ſo, as that the two points of the compaſſes may reach (croſs-wiſe on the ſector) from 60 to 60 degrees in the line of the chords; and keeping the ſector at that opening, take off 4 degrees and 93 hundredth parts of a degree (as near as you can gueſs by the eye) croſs-wiſe, from the line of chords, near their beginning at the joint; and ſet that ex-

tent

tent with your compaſſes upon the periphery of the circle, making marks with the points, and divide the ſpace between the marks into 5 equal parts; and then divide the reſt of the circle, firſt into ſix equal parts, then each of theſe again into ſix, and each of theſe laſt into ten; and ſo you will have the whole circle divided into 365 equal parts, as was required.

Again, ſuppoſe a given circle was to be divided into 59 equal parts: ſubtract 9, and there will remain 50. Then, as 59 parts are to 360 degrees, ſo are 9 parts to the meaſure of the arc they will contain; which, by the operation, will be found to be 54.91 degrees. Therefore, ſet off 54.91 (or $54\frac{9}{10}$) degrees upon the circle, and divide that ſpace into 9 equal parts; then divide the reſt of the circle, firſt into 5 equal parts, and then each of theſe parts into 10; and the whole will be divided into 59 equal parts, as was required. As twice $29\frac{1}{2}$ make 59, this diviſion will do very well for a circle containing the

the 29½ days of the Moon's age from change to change; as the above division of 365 will do for the days in a common year.

Once more; ſuppoſe a given circle muſt be divided into 31 equal parts: ſubtract 1, and there will remain 30. Then, as 31 parts are to 360 degrees, ſo is one part to 11.61 degrees (or 11.6) near enough for the purpoſe. Therefore, ſet off 11.6 degrees on the circle for the one odd part; and divide the reſt of the circle, firſt into ſix equal parts, and then each of theſe parts into five: and the whole circle will be divided into the required number of 31 equal parts. This anſwers to the diviſions of the common month-day circle in clocks.

The method of dividing ſtraight lines of given lengths into any given number of equal parts is the ſame as above; only, inſtead of degrees and parts of a degree, we make uſe of inches and parts of an inch. Thus,

Suppoſe

Suppose a given straight line was 7 inches and $\frac{3}{10}$ parts of an inch (or 7.3 inches) in length; and it was required to divide that line into 43 equal parts. Subtract 3, and there will remain 40: then, as 43 parts is to 7.3 inches, so is 3 parts to a fourth proportional number, which will be found to be 0.51, or $\frac{51}{100}$ part of an inch. Therefore, from a common scale, where you have an inch diagonally divided into 100 equal parts, take off 51 of these parts in your compasses; and setting one foot in either of the ends of the given line, make a mark with the other foot upon the line; and divide that space into 3 equal parts, and the rest of the line into 40; and you will have the whole line divided into 43 equal parts, as was desired.

By this method, a wheel may be easily divided into any given number of teeth. For, if a slip of paper be laid round the edge of the wheel, so as the ends may just meet; the said slip may be divided into as many equal parts as the wheel must have teeth; and then, having

having put a little ſtarch or gum all round the edge of the wheel, put the paper round it again, and divide the wheel by the marks on the paper.

How to find two requiſite points in the tube of a Thermometer, and then to divide the ſcale thereof.

First, put the bulb of the tube into water juſt freezing, or ſnow juſt thawing; and, at that part where the top of the Mercury ſettles in the tube, make a mark on the tube. Then, if the tube be long enough to contain the riſing of the Mercury for boiling water, put the bulb into boiling water; and, at the height to which the boiling raiſes the Mercury, make a mark on the tube. This done, apply the tube to the intended ſcale; and againſt the firſt mark, place the number 32 on the ſcale, to denote the freezing point; and againſt the ſecond mark, place the number 212 on the ſcale, to denote the boiling point.

point. This done, divide the ſpace between the two marks on the ſcale into 180 equal parts; and continue theſe diviſions both above the boiling and below the freezing point, for the whole length of the tube upon the ſcale, and number them accordingly. And then the heats of ſeveral bodies may be ſhewn, by being expreſſed on the ſcale, as follows.

Air, in ſevere cold weather in Britain, from 15 to 25. Air at Midſummer, from 65 to 68. Extreme heat of the Summer Sun, from 86 to 100. Human heat, about 97. Butter juſt melting, 95. Alcohol (or pure ſpirits) boils with 174 or 175, Brandy with 190, water with 212, oil of Turpentine with 550, and Tin melts with 408. Milk freezes about 30, Vinegar about 28, and Blood about 27.

If the tube be not long enough to bear boiling water, firſt find the freezing point as above directed; and then put the bulb under your arm-pit, next to your ſkin, and hold it there as long as

 you

you obſerve the Mercury to riſe in the tube, and then make a mark on the tube where the Mercury ſettles. This done, apply the tube to the intended ſcale; on which, place 32 at the firſt mark, for the freezing point, and 97 at the ſecond mark for the degree of human heat. Then divide the ſpace between theſe two numbers on the ſcale, into 65 equal parts, and continue theſe diviſions above 65, and below 32, as far as the tube goes; and laſtly, place the different appellations of heat and cold on the ſcale as abovementioned.

Rules for finding the Areas or ſuperficial contents of Plane Figures and of ſolid Bodies.

1. *The diameter of a circle being given, to find its circumference and its area.*

As 113 is to 355, ſo is the diameter of the circle, taken in any meaſure, as

as feet, inches, &c. to its circumference in the like meaſures.

To find the area, in circular meaſure, multiply the diameter by itſelf, and the product will be the area ſought.

To find the area in ſquare meaſure, ſay, as 1 is to 0.7854 (or rather 0.785399) ſo is the ſquare of the diameter to the area ſought, in ſuch meaſures as the diameter was taken.

2. *To find the area of the ſector of a Circle.*

As 365 degrees is to the degrees in the arc of the ſector, ſo is the area of the whole circle to the area of the ſector. Or, multiply the length of the radius of the ſector, by the length of the arc of the circle, under which it is contained, and divide the product by 2; the quotient is the area ſought.

3. *To find the area of an Ellipſis.*

Multiply the longeſt diameter by the ſhorteſt, and the product by 0.7854;

the laſt product is the area ſought, in ſquare meaſure.

4. *To find the area of a Parabola.*

Multiply the baſe, or greateſt ordinate, by the perpendicular height, and the product by 2: then divide the laſt product by 2, and the quotient will be the area required.

5. *To find the area of a Square, or of a Parallelogram.*

Multiply any ſide of the ſquare by itſelf, or one of the longeſt ſides of the parallelogram by one of the ſhorteſt; and the product will be the area required.

6. *To find the area of a Triangle, the lengths of whoſe ſides are given.*

If the triangle be a right angled one, multiply the baſe thereof by the perpendicular,

pendicular, and divide the product by 2; the quotient will be the area, in ſuch ſquare meaſures as the lengths of the baſe and perpendicular were given.

If the triangle be an oblique one, and have its ſides of different lengths, call the longeſt ſide the baſe; and, from the angular point, oppoſite to the baſe, draw a right line perpendicular to the baſe. Then, find the length of the perpendicular, in ſuch meaſures as the lengths of the ſides were given, and multiply the length of the baſe by the length of the perpendicular, and divide the product by 2: the quotient will be the area of the whole triangle, as was required.

7. *To find the area, or ſuperficial content, of any Rectilineal Figure.*

If the figure be irregular, and conſiſts of many ſides and angles, reduce it all into triangles; and then find the content of each triangle by itſelf, as directed by the foregoing rule. The ſum of

of all theſe areas or contents, being added together, will be the area or content of the whole figure.

8. *To find the content of a Field that has been ſurveyed by the chain, and plotted down by a ſcale of chains and links.*

The common chain for ſurveying contains 100 links, and the common plotting ſcale has inches divided by diagonal lines into 100 equal parts; ſo that an inch anſwers to a chain in plotting, and the parts to links. 'Tis all the ſame if half an inch, or a quarter of an inch, be divided into 100 equal parts; for they may be uſed in plotting: and always the ſmalleſt diviſions are uſed when the largeſt quantities of land are to be plotted down on paper.

The field being plotted down, reduce it all into triangles; and meaſure the baſe and perpendicular height of each triangle, by your compaſſes, on the ſcale by which the field was plotted, ſetting down the number of chains and links

links of each meaſure as if they were all whole numbers of links. Thus 7 chains, 24 links, are wrote down, 724 links, and 5 chains, 9 links, are wrote down 509 links, and ſo on. Then multiply the length of the baſe of each triangle in links, by its perpendicular height in links, and add all the products together into one ſum: which done, divide the whole ſum by 2, and the quotient will be the area of the field, in ſquare links of the chain.

From the number of ſquare links contained in this area, cut off five figures to the right hand (becauſe 100000 ſquare links make an acre) and what remains to the left hand will be Acres; and thoſe which are cut off will be decimal parts of an Acre.

Multiply theſe decimals of an Acre by 4, and from the product cut off five figures to the right hand; and what remains to the left will be Roods, as thoſe cut off will be decimals of a Rood.

Laſtly, multiply theſe decimals of a Rood by 40, and cut off five figures of the

the product to the right hand: and what remains to the left will be Perches, as thoſe cut off will be decimals of a Perch.

And, by this method, the whole content of a field will be had, in acres, roods, and Perches.

9. *The diameter of a Circle being given, to find the ſide of a ſquare whoſe area is equal to the area of the Circle: and the reverſe.*

As 1 is to 0.8862, ſo is the diameter of the circle to the ſide of the ſquare, whoſe area is equal to the area of the circle. Or having found the area of the circle by Prob. 1. extract the ſquare root of that area, and it will be the ſide of the ſquare ſought.

To find the diameter of a circle whoſe area ſhall be equal to the area of a given ſquare; ſay, as 1 is to the ſide of the ſquare, ſo is 1.128 to the diameter of the circle required.

10. *To*

10. *To find the area or ſuperficial contents of a Globe.*

As 113 is to 355, ſo is the ſquare of the diameter of the globe to its ſuperficial content.

11. *To find the ſuperficial content of a Cylinder.*

As 1 is to the length of the cylinder, ſo is its circumference to the ſuperficial content of its convexity, to which add the areas, or two flat circular ſurfaces at each end (found by Prob. 1), and the whole will be the ſuperficial content required.

12. *To find the ſuperficial content of a Cone.*

As 1 is to the oblique (not the perpendicular) height of the cone, ſo is

 half

half the circumference of the baſe to the ſuperficial content of the convexity; to which add the area, or ſuperficial content of the baſe, and you will have the whole ſuperficial content of the cone, as was required.

13. *To find the ſuperficial content of the Fruſtum of a Cone.*

As 1 is to the oblique altitude of the Fruſtum, ſo is half the ſum of the circumference, at the top and bottom, to the ſuperficial content of the convex part: to which add the ſuperficial contents of the circular top and baſe, and you will have the whole ſuperficiality of the Fruſtum required.

14. *To find the ſuperficial content of a Priſm.*

This may be found in all reſpects as in the Cylinder, regard being only had to the figure of the baſe.

15. *To*

15. *To find the superficial content of a Pyramid.*

This is done in all reſpects as the cone, regarding the figure of the baſe: and conſequently the ſuperficial content of the Fruſtum of a Pyramid may be found in the ſame manner.

16. *To find the ſuperficial content of any of the five Platonic bodies.*

As 1 is to the ſide of the given Platonic body,

ſo is	1.73205	To the ſuperficial content of the	Tetrëdron.
	3.46410		Oċtaëdron.
	6.00000		Hexaëdron.
	8.66025		Icoſaëdron.
	20.64573		Dodecaëdron.

Or, as 1 is to the ſquare of the ſide of either of theſe Platonic bodies, ſo are the above numbers in this propoſition, to the ſuperficial content of the reſpective Platonic body.

16. *The diameter of a ſphere being given, to find the ſide of any of the Platonic bodies, that may be either inſcribed in the ſphere, or circumſcribed about the ſphere, or that is equal to the ſphere.*

As 1 is to the number in the following Table, reſpecting the thing ſought, ſo is the diameter of the given ſphere to the ſide of the Platonic body ſought.

The diameter of a ſphere being unity, the ſide of a	That may be inſcribed in the ſphere, is	That may be circumſcribed about the ſphere, is	That is equal to the ſphere, is
Tetraëdron	0.816497	2.44948	1.64417
Octaëdron	0.707107	1.22474	1.03576
Hexaëdron	0.577350	1.00000	0.80610
Icoſaëdron	0.525731	0.66158	0.62153
Dodecaëdron	0.356822	0.44903	0.40883

17. *The ſide of any of the five Platonic bodies being given, to find the diameter of a ſphere that may be inſcribed in that body, or circumſcribed about it, or that is equal to it.*

As the reſpective number in the above Table, under the title, *Inſcribed*, *Circum-*

Circumſcribed, Equal, is to 1; ſo is the ſide of the given Platonic body, to the diameter of its inſcribed, circumſcribed, or equal ſphere, in ſolidity.

18. *The ſide of any of the five Platonic bodies being given, to find the ſide of either of the Platonic bodies which are equal in ſolidity to that of the given body.*

As the number under the title *Equal,* againſt the given Platonic body, is to the number under the ſame title againſt the body whoſe ſide is ſought, ſo is the ſide of the given Platonic body to the ſide of the Platonic body ſought.

Rules for finding the ſolid contents of Bodies.

1. *To find the ſolid contents of a Sphere or Globe.*

Multiply the diameter of the ſphere twice into itſelf (which is cubing it) and

and the Product by 0.5236; the laſt product is the ſolidity required.

2. *To find the ſolid content of a Spheroid.*

As 14 is to 11, ſo is the ſquare of the conjugate diameter, multiplied by two thirds of the tranſverſe diameter, to the ſolid content required.

3. *To find the ſolid content of a Cube.*

Multiply the ſide of the cube twice into itſelf, and the product will be the ſolid contents thereof.

4. *To find the ſolid contents of a Parallelipidon.*

Multiply the length by the breadth, and the product by the depth; the laſt product will be the ſolid content required.

5. *To find the ſolid contents of a Priſm.*

Multiply the area of the triangular baſe by the height of the Priſm; and the

the product will be the solid contents thereof.

6. To find the solid contents of a Cone, and also of a Pyramid.

As 3 is to the area of the base, taken in any measure, so is the perpendicular altitude of the Cone, or of the Pyramid, to its solid contents, in the same measure.

7. To find the solid contents of the Frustum of a Cone.

First find the areas of its circular base and top: then multiply these areas into one another, and extract the square root of the product: add this square root to the sum of the foresaid areas, and multiply this whole sum by one third of the perpendicular height of the Frustum, and the product will be the solid content thereof.

8. *To*

8. *To find the ſolidity of the Fruſtum of a Pyramid.*

This is done in all reſpects as in the Fruſtum of a Cone, only having reſpect to the figures of its flat baſe and top, as they may be triangular, ſquare, hexagonal, &c.

9. *To find the ſolid contents of any of the five Platonic bodies.*

As 1 is to the cube of the ſide of any of theſe bodies, ſo is 0.117851 to the ſolid contents of the Tetraëdron, 0.417404 to that of the Octaëdron, 1.00000 to that of the Hexaëdron, 2.181695 to that of the Icoſaëdron, and 7.663199 to the ſolid content of the Dodecaëdron.

10. *To find the ſolid contents of any irregular body, even if it were a Gooſeberry buſh, provided you have a veſſel that will fully hold it.*

Let the veſſel be filled quite up to the brim with water, and weighed in a balance:

balance: then put the irregular body into the veſſel, till it be quite covered with the water, and it will cauſe as much water to run over, as is equal to its whole bulk. This done, take the body out of the water, and then find how much leſs the veſſel weighs, than it did, when full of water, before the body was put into it. Reduce this deficiency of weight into Troy grains, and divide the number of grains by 253.18287 (becauſe the weight of a cubic inch of common water is 253.18287 grains) and the quotient will be the ſolid contents of the body in cubic inches, which may be reduced to cubic feet by dividing the number of inches by 1728, the number of cubic inches in a cubic foot.

N. B. The outſide of the veſſel muſt be wetted when it is full of water, and its weight taken, before the body be put into it; for otherwiſe, part of the water which the body cauſes to run over, when it is immerſed, will

ſtick to the outſide of the veſſel, and thereby give a falſe concluſion.

11. *To find the ſolid contents of a Cylinder, in cubic inches.*

As 1 is to 0.7854 (or rather to 0.785399) ſo is the ſquare of the diameter of the Cylinder, taken in inches, to the number of ſquare inches contained in the area of the baſe of the Cylinder: which number being multiplied by the height of the Cylinder, taken in inches, gives the ſolid content thereof in cubic inches.

Now, ſuppoſing the Cylinder to be hollow, and theſe meaſures to be taken in the inſide; we may find how much it will hold, in Ale gallons, Wine gallons, Corn gallons, or Corn buſhels, thus:

Divide the content in cubic inches by 282, and the quotient will be the number of Ale gallons; by 231, and the quotient will be the content in Wine gallons; by 268,8, and the quotient will

will be Corn gallons; and by 2150.42, and the quotient will be the content in Corn buſhels.

12. *To gauge a common Caſk or Barrel.*

Meaſure the inſide diameters of the Caſk at the end and middle, and take their difference in inches. Multiply this difference by 0.7, and add the product to the diameter at the end; which will give the mean diameter of the caſk, very nearly, as if it were a cylindrical veſſel of the ſame contents with that of the caſk: which contents may be found in Ale gallons, Wine gallons, Corn gallons, or Corn buſhels, by the foregoing Problem.

13. *To gauge a common Vat.*

Multiply the diameters at the top and bottom (taken in inches) into one another, and to their product add one third part of the ſquare of their difference; then multiply this ſum by 0.7854, and

and the product will be a mean area, as if the veſſel was cylindrical. Multiply this area by the perpendicular height of the Vat, and the product will give the contents thereof, in the number of cubic inches that it will hold: which number may be reduced into Ale gallons, Wine gallons, Corn gallons, or Corn buſhels, as above.

A Table, by which the quantity and weight of water in a cylindrical pipe of any given diameter of bore, and perpendicular height, may be found: and conſequently, the power may be known that will be ſufficient to raiſe the water to the top of the pipe, in any pump, or other Hydraulic machine.

Feet high.	Diameter of the cylindric bore 1 Inch.		
	Quantity of water in cubic Inches.	Weight of water in Troy ounces.	In Avoirdupoiſe ounces.
1	9.4247781	4.9712340	5.4541539
2	18.8495562	9.9424680	10.9083078
3	28.2743343	14.9137020	16.3624617
4	37.6991124	19.8849360	21.8166156
5	47.1238905	24.8561700	27.2707695
6	56.5486686	29.8274040	32.7249234
7	65.9734467	34.7986380	38.1790773
8	75.3982248	39.7698720	43.6332312
9	84.8230029	44.7411060	49.0873851

For

For tens of feet high, remove the decimal points one place forward; for hundreds of feet, two places; for thousands, three places; and so on. Then multiply the sums by the square of the diameter of the given bore, and the products will be the anſwer.

EXAMPLE.

Qu. *The quantity and weight of water in a cylindrical pipe 85 feet high, and 10 inches diameter.* The ſquare of 10 is 100.

Feet high.	Cubic inches.	Troy ounces.	Avoird. ounces.
80	753.982248	397.698720	436.332312
5	47.123890	24.856170	27.270769
85	801.106138	422.554890	463.603081
	mult. by 100	100	100
Anſ.	80110.613800	42255.489000	46360.308100

Which number (80110.6) of cubic inches being divided by 231, the number of cubic inches in a Wine gallon, gives 342.6 for the number of gallons: and the reſpective weights (42255.489, and

and 46360.3) being divided, the former by 12, and the latter by 16, give 3521.29 for the number of Troy pounds, and 2897.5 for the number of Avoirdupoise pounds, that the water in the pipe weighs. So much power would be required to balance or support the water in the pipe, and as much more to work the engine as the friction thereof amounts to.

Concerning Pumps.

In all Pumps, the pressure of the column of water, or its weight felt by the working power, when raised to any given height above the surface of the well, is in proportion to the height of the column, considered throughout, as if it were equal in diameter to that part of the bore in which the piston or bucket works.

The advantage or power gained by the handle of the pump is the same as in the common lever; that is, as great as the length from the axis of the handle

to

to its end where the power is applied, exceeds the length of the other part of the handle, from the axis on which it turns, to the pump rod wherein it is fixed, for lifting the piſton and water.

In the making of pumps, the diameter of the bore where the bucket works ſhould be proportioned to the height which the pump raiſes water above the ſurface of the well, as that a man of ordinary ſtrength might work all pumps equally eaſy, let their heights be what they will. The annexed Table ſhews how this may be done, and what quantities of water may be raiſed in a minute by one man, ſuppoſing the handle of the pump to be a lever increaſing the power five times.

N. B. In my book of Lectures, pag. 75, laſt paragraph, and line 3 of column 1, in pag. 76, *for* bucket, *read* ſurface of the water in the well.

A Table

Find the given height of the pump, in the first column of the Table; and against it in the second column, you have the diameter which the bore must be of, in inches and hundredth parts of an inch: 2nd in the third column, you have the quantity of water, in English gallons and pints, that a man of common strength can raise to that height in a minute.

With respect to the power required to work the pump, or the quantity of water discharged thereby, it matters not what the diameter of the bore be in any other part than that wherein the piston or bucket works.

A Table for Pump-makers.

Height of the pump above the surface of the well.	Diameter of the bore where the Piston works.	Water discharged in a minute in gallons and pints.	
Feet.	Inches.	Gal.	Pints.
10	6.93	81	6
15	5.65	54	4
20	4.90	40	7
25	4.38	32	6
30	4.00	27	2
35	3.70	23	3
40	3.47	20	4
45	3.26	18	1
50	3.10	16	3
55	2.95	14	7
60	2.83	13	5
65	2.71	12	4
70	2.62	11	5
75	2.53	10	7
80	2.44	10	2

Troy weight compared with Avoirdupoise weight.

175 Troy pounds are equal to 144 Avoirdupoise pounds.
175 Troy ounces are equal to 192 Avoirdupoise ounces.
1 Troy pound contains 5760 grains; and
1 Avoirdupoise pound contains 7000 grains.
1 Troy ounce contains 480 grains; and
1 Avoirdupoise ounce contains 437.5 grains.
1 Avoirdupoise dram contains 27.34375 grains.
1 Troy pound = 13 ounces 2.651428576 Avoird. drams; and
1 Avoirdupoise pound is equal to 1 pound, 2 ounces, 11 penny weight, 16 grains Troy.

By

By the following Table, we may find how much of either of theſe weights is contained in any given number of pounds in the other.

Tr. P.	Avoird. pounds.	Av. P.	Troy pounds.
1	0.822857142857143	1	1.215277777777778
2	1.645714285714286	2	2.430555555555556
3	2.468571428571429	3	3.645833333333333
4	3.291428571428572	4	4.861111111111111
5	4.114285714285715	5	6.076388888888889
6	4.937142857142857	6	7.291666666666667
7	5.760000000000000	7	8.506944444444444
8	6.582857142857143	8	9.722222222222222
9	7.405714285714286	9	10.937500000000000

For tens of pounds, remove the decimal points one place forward; for hundreds of pounds, two places; for thouſands, three places; for tens of thouſands, four places; and ſo on, as in the following Examples.

When any fractions remain in the laſt ſum, reduce them to the known parts of a pound, by the common method of reducing decimals to the known parts of an integer: remembering, that in Troy weight, 12 ounces make a pound, 20 penny weight make an

 ounce,

ounce, and 24 grains make a penny weight: and that, in Avoirdupoise weight, 16 ounces make a pound, and 16 drams make an ounce.

Example I.

In 175 *Troy pounds,* Qu. *How many Avoirdupoise pounds?*

Troy.	Avoirdupoise.	
100	82.2857142857143	
70	57.6000000000000	*Answ.* 144.
5	4.1142857142857	
175	144.0000000000000	

Example II.

In 144 *Avoirdupoise pounds,* Qu. *How many Troy pounds?*

Avo.	Troy.	
100	121.527777777778	
40	48.611111111111	*Answ.* 175.
4	4.861111111111	
144	175.000000000000	

Exampl.

EXAMPLE III.

In 72 Avoirdupoiſe pounds, Qu. How much Troy weight?

Avo.	Troy.	
70	85.0694444444444	*Anſwer.*
2	2.4305555555556	87.5 pounds,
		viz. 87 pounds
72	87.5000000000000	6 ounces.
P.	P.	

In common practice, 'tis ſufficient to take out the decimal part to five or ſix figures.

By four weights, viz. 1 *pound,* 3 *pounds,* 9 *pounds, and* 27 *pounds, to weigh* 40 *pounds; or any number of pounds from* 1 *to* 40.

Pounds.	Scale A.	Scale B	Pounds.	Scale A.	Sc. B.
1	1	0	21	27,3	9
2	3	1	22	27,3,1	9
3	3	0	23	27	3,1
4	1,3	0	24	27	3
5	9	3,1	25	27,1	3
6	9	3	26	27	1
7	9,1	3	27	27	0
8	9	1	28	27,1	0
9	9	0	29	27,3	1
10	9,1	0	30	27,3	0
11	9,3	1	31	27,3,1	0
12	9,3	0	32	27,9	3,1
13	9,3,1	0	33	27,9	3
14	27	1,3,9	34	27,9,1	3
15	27	3,9	35	27,9	1
16	27,1	3,9	36	27,9	0
17	27	9,1	37	27,9,1	0
18	27	9	38	27,9,3	1
19	27,1	9	39	27,9,3	0
20	27,3	9,1	40	27,9,3,1	0

The two columns under *Pounds* expreſs the number of pounds to be weighed: to the right hand of which, the column under *A* ſhews what weights are to be put into one ſcale of the balance, and the column under *B* ſhews what weights are to be put into the other: by which means, the ſcale *B* will be ſo much lighter than the ſcale *A*, as to require a weight to be put into it, equal to the given number of pounds to be weighed, as ſtated at the left hand, under *Pounds*, againſt the weights in the ſcales; and then the balance will be even.

If, to the above four weights, one of 81 pounds be added, you may weigh 121 pounds; or any number from 1 to 121.
to 121: and if, to theſe, you add a weight of 243 pounds, you may weigh 364 pounds, or any number from 1 to 364.

A Table

A Table of the ſpecific gravities of Bodies.

A cubic inch of	Troy Weight. oz.	pw.	gr.	Avoird. oz.	drams	Comparative weight.
Very fine Gold	10	7	4.45	11	5.85	19.639
Standard Gold	9	19	6.06	10	14.88	18.887
Guinea Gold	9	7	17.18	10	4.76	17.793
Quickſilver	7	3	4.39	7	13.16	13.565
Lead	5	19	16.32	6	8.86	11.325
Pure Silver	5	17	0.00	6	6.69	11.090
Standard Silver	5	11	3.25	6	1.55	10.534
Copper	4	14	22.62	5	3.33	9.000
Plate Braſs	4	8	2.05	4	13.31	8.344
Caſt Braſs	4	5	10.76	4	10.08	8.001
Steel	4	2	20.21	4	8.71	7.835
Block Tin	3	17	5.52	4	3.99	7.320
Diamond	1	15	21.07	1	13.35	3.400
Fine Marble	1	8	14.11	1	9.30	2.710
Common Glaſs	1	7	5.20	1	7.88	2.579
Alabaſter	0	19	18.43	1	2.03	1.873
Dry Ivory	0	19	5.83	1	0.89	1.823
Dry Boxwood	0	10	20.77	0	9.54	1.201
Sea Water	0	10	20.51	0	9.51	1.035
Common Water	0	10	13.18	0	9 23	1.000
Red Wine	0	10	11.42	0	9.20	.993
Proof Spirits	0	9	19.73	0	8.62	.931
Pure Spirits	0	9	3.27	0	8 02	.866
Æther	0	7	14.00	0	7.46	.720
Cork	0	2	12.77	0	2.21	.240
Air	0	0	0.25	0	0.009	.001

Take away the decimal points from the numbers in the right hand column, and reckon them to be whole numbers; and they will ſhew how many Avoirdupoiſe ounces are contained in a cubic foot of each of the above bodies in the Table.

A Table

A Table of the different Velocities and Forces of the Winds.

Velocity of the Wind. Miles in one Hour.	= Feet in one Second.	Perpendicular force on one foot Area, in pounds Avoirdupoise.	Common appellations of the forces of Winds.
1	1.47	.005	Not perceptible.
2	2.93	.020	Just perceptible.
3	4.40	.044	
4	5.87	.079	Gentle pleasant Wind.
5	7.33	.112	
10	14.67	.492	Pleasant brisk Gale.
15	22.00	1.107	
20	29.33	1.968	Very brisk.
25	36.67	3.075	
30	44.00	4.428	High Winds.
35	51.33	6.027	
40	58.67	7.872	Very high.
45	66.00	9.963	
50	73.33	12.300	A storm or tempest.
60	88.00	17.712	A great storm.
80	117.33	31.488	A hurricane.
100	146.70	49.200	A hurricane that tears up trees, and carries buildings, &c. before it.

The force of the Wind is as the square of its velocity.

That the force of the wind is as the square of its velocity, I have often proved by experiments made on my *Whirling Table.*

Of

Of the difference between the apparent Level and the true.

When a plumb line hangs freely, it hangs directly toward the center of the earth: and a right line, crossing the direction of the plumb line at right angles, and touching the Earth's surface just below the plummet, is a level at that point of the Earth's surface. But, if this right line be continued from that point, keeping still perpendicular to the plumb line, it will rise above the Earth's surface, because the Earth is of a globular shape: and this rising will be as the square of the distance to which the said right line is produced. That is, however much it rises above the surface at one mile's distance, it will rise four times as much at the distance of two miles; nine times as much at the distance of three miles; sixteen times as much at the distance of four miles; and so on. And therefore, if two levels are taken at two points of the Earth's

Earth's ſurface which are at any conſiderable diſtance (as ſuppoſe a mile) from each other, the level lines produced will interſect each other at a certain angle: and although either of them, ſo produced, will appear to be a true level, yet it can be ſo only at that point of the Earth's ſurface from which it was produced: not at any other.

The height to which a level line, produced from any given place, riſes above any other place, is the height of the apparent level above the true at that other place: the quantity of which height is ſhewn by the following Table, for all diſtances within the length of a degree of a great circle upon the Earth's ſurface. There is a Table of the ſame ſort in Dr. LONG's Aſtronomy, which differs but two inches from this (which I have computed) in the height of the apparent level above the true, for a whole degree, or 60 geographical miles; which are longer than 60 Engliſh miles by 4884o feet.

By

By the moſt accurate meaſures of the length of a degree on the Earth's ſurface, the whole 360 degrees of the Earth's circumference contain 131630400 feet, or 24930 Engliſh miles: which in geographical miles (allowing 60 to a degree of a great circle) make only 21600. So that, a geographical mile contains 6094 feet, which exceeds the length of an Engliſh mile by 814.

In the Table, a geographical mile (which I have often thought ſhould be the univerſal ſtandard length of a mile) is called a *minute*, becauſe it is the 60th part of a degree; and the 60th part of ſuch a mile is called a *ſecond*.

As the ſurface of water naturally anſwers to the curvature of the Earth's ſurface (ſuppoſing no hills or eminencies thereon) 'tis plain that if a long ſtraight channel was made, ſo as to have its middle part level at any part of the Earth's ſurface, and the reſt continued out both ways in direction of an apparent level from that place; if water ſhould come in at either end of the

channel, it would run to the middle thereof: and, if the channel was all of an equal depth, the water would run over at the middle before the channel could be filled at both its ends. And consequently, if a distant spring appeared by a levelling instrument to be just on a level with the house, the water might be brought in a straight channel from the spring to the house; or in pipes, if there was an intervening valley; because the water will rise in crooked pipes, till its surface at both ends is equally distant from the Earth's center.

A Table shewing the height of the apparent level above the true, to the 1000dth part of an Inch.

Seconds.	Feet.	Inches.	Inches.
If the distance of the object from the place of the spectator be	which measured in a great circle upon the Earth's surface amounts to		the height of the apparent level above the true will be
1	101	6.8	0.003
2	203	1.6	0.012
3	304	8.4	0.027
4	406	3.2	0.048
5	507	10.0	0.074
6	609	4.8	0.106
7	710	11.6	0.145
8	812	6.4	0.189
9	914	1.2	0 239
10	1015	8.0	0.295
11	1117	2.8	0.357
12	1218	9.6	0.425
13	1320	4.4	0.499
17	1421	11.2	0.579
15	1523	6.0	0.665
16	1625	0.8	0.756
17	1726	7.6	0.854
18	1828	2.4	0.947
19	1929	9.2	0.067
20	2031	4.0	1.182
21	2132	10.8	1.303
22	2234	5.6	1.420
23	2336	0.4	1.563
24	2437	7.2	1.702
25	2539	2.0	1.847
26	2640	8.8	2.001
27	2742	3.6	2.154
28	2843	10.4	2.316
29	2945	5.2	2.485
30	3047	0.0	2.659

The Table continued.

If the diſtance of the object from the place of the ſpectator be | which meaſured in a great circle upon the Earth's ſurface amounts to | the height of the apparent level above the true will be

Seconds.	Feet.	Inches.	Inches.
31	3148	6.8	2.839
32	3250	1.6	3.026
33	3351	8.4	3.218
34	3453	3.2	3.416
35	3554	10.0	3.619
36	3656	4.8	3.829
37	3757	11.6	4.045
38	3859	6.4	4.267
39	3961	1.2	4.494
40	4062	8.0	4.728
41	4164	2.8	4.967
42	4265	9.6	5.212
43	4367	4.4	5.463
44	4468	11.2	5.720
45	4570	6.0	5.983
46	4672	0.8	6.252
47	4773	7.6	6.527
48	4875	2.4	6.808
49	4976	9.2	7.094
50	5078	4.0	7.387
51	5179	10.8	7.685
52	5281	5.6	7.989
53	5383	0.4	8.300
54	5484	7.2	8.616
55	5586	2.0	8.938
56	5687	8.8	9.266
57	5789	3.6	9.600
58	5890	10.4	9.940
59	5992	5.2	10.285
60	6094	0.0	10.637

The Table continued.

Minutes.	Feet.	Feet.	Inches.
If the distance of the object from the place of the spectator be	which measured in a great circle upon the Earth's surface amounts to	the height of the apparent level above the true will be	
1	6094	0	10.637
2	12188	3	6.548
3	18282	7	11.732
4	24376	14	2.191
5	30470	22	1.923
6	36564	31	10.929
7	42658	43	5.209
8	48752	56	8.763
9	54846	71	9.591
10	60940	88	7.692
11	67034	107	3.067
12	73128	127	7.716
13	79222	149	10.025
14	85316	173	8.836
15	91410	199	5.307
16	97504	226	11.052
17	103598	256	2.070
18	109692	287	2.362
19	115786	319	11.928
20	121800	354	6.768
21	127974	390	10.882
22	134068	429	0.269
23	140162	468	10.931
24	146256	510	6.866
25	152350	554	0.075
26	158444	599	2.558
27	164538	646	2.315
28	170632	694	11.345
29	176726	745	5.649
30	182820	797	9.928

The

The Table concluded.

Minutes.	Feet.	Feet.	Inches.
If the distance of the object from the place of the spectator be	which measured in a great circle upon the Earth's surface amounts to	the height of the apparent level above the true will be	
31	188914	851	10 080
32	195008	907	8.206
33	210102	965	3.606
34	207196	1024	8.280
35	213290	1085	10.227
36	219384	1148	9.448
37	225478	1213	5.493
38	231572	1279	11.712
39	237666	1348	2.755
40	243760	1418	3.072
41	249854	1490	0.663
42	255948	1563	7.527
43	262042	1638	11.665
44	268136	1716	1.077
45	274230	1794	11.763
46	280324	1875	7.723
47	686418	1958	0.956
48	292512	2042	3.464
49	298606	2128	3.245
50	304700	2216	0.300
51	310794	2305	6.629
52	316888	2396	10.232
53	322982	2489	11.108
54	329076	2584	9.259
55	335170	2681	4.683
56	341264	2779	9.381
57	347358	2879	11.353
58	353452	2981	10.598
59	359546	3085	7.199
60	365640	3191	0.912

By

By the preceding Table, the length of an arc, in feet and inches, on the Earth's ſurface, may be found; if its meaſure be known in minutes and ſeconds of a degree. Thus, ſuppoſe the length of the arc be 10 ſeconds, which is the ſixth part of a geographical mile; its meaſure is 1015 feet, 8 inches: and an arc of one minute of a degree, which is one geographical mile, is found to be 6094 feet.

We may alſo find how far one can ſee in a true horizon at ſea, when the eye is raiſed to any given height above the ſurface of the water. Thus, ſuppoſe the eye of an obſerver on a ſhip at ſea to be 22 feet, 2 inches, above the ſurface of the ſea, he will ſee to the diſtance of 30470 feet all around him; or to the diſtance of 5 geographical miles: for againſt 22 feet, 1.923 (which may be eſteemed 2) inches, in the right hand column, is 30470 feet, in the middle column; and 5 minutes or geographical miles in the firſt.

Again,

Again, ſuppoſing that a gentleman was to make a canal in a piece of ground which is half a geographical mile long, and appeared to be truly level by the common levelling inſtrument; this being 30 ſeconds of a degree, or 3047 feet, in length, the height of the apparent level above the true, at that diſtance, is 2.659 inches: and ſo much muſt the farther end of the ground be ſunk, in order to have the water equally near the ſurface of the ground at both ends of the canal.

Once more; ſuppoſe an obſerver to have his eye cloſe at the ſurface of the ſea, and that he then juſt ſees the top of a mountain in the ſea, whoſe diſtance he knows to be juſt 60 geographical miles, or 365640 feet; the perpendicular height of that mountain, above the ſurface of the ſea, is 3191 feet and $\frac{9}{10}$ parts of an inch.

If the diſtance of the mountain ſo ſeen be more than 60 geographical miles, which is without the reach of the

of the Table; yet its height may be found by the Table, in the following manner.

Suppose the distance of the mountain to be 90 geographical miles, or a degree and an half, in a great circle upon the Earth: take half the given number of miles or minutes, which is 45; and multiply the height of the apparent level above the true, at that distance, by 4: and the product will give the perpendicular height of the mountain. Thus, against 45 minutes you have the height 1794 feet, 11.763 inches; which multiplied by 4, gives 7917 feet, 11.052 inches, for the height of the mountain above the surface of the sea.

According to the measures in the Table, a degree of a great circle upon the Earth contains 69¼ English miles.

Of the Mechanical Powers, and of Friction.

From the most simple machine to the most compound Engine, the power or

advantage gained is always as much as the velocity of the moving power exceeds the velocity of the weight or resistance that is moved; making proper allowance for the friction of the machine or engine. So that, if the working power moves through a space of ten, or an hundred, or a thousand inches, whilst the weight or resistance moves only through the space of one inch; the person who works the machine or engine (supposing it to have no friction) could raise ten, or an hundred, or a thousand times as much weight as he could do by his natural strength without it. But the time that is lost will be always as much as the power that is gained.

The simple machines by which power is gained, are six in number; *viz.* the *Lever*, the *Wheel and Axle*, the *Pullies*, the *Inclined Plane*, the *Wedge*, and the *Screw*. Of these six simple machines, all the most compound engines are made: for we know of no other simple machines by which power can be gained.

1. A lever

1. A lever is a bar laid over any prop that will ſupport the weight upon it. If the prop be under the middle of the lever, no advantage is gained by it; for, as faſt as a man puſhes down one end, the power riſes on the other. To gain power by it, the length of the part or arm between the man and the prop muſt be greater than the length of the part or arm between the prop and the weight. And then, as much power will be gained as the length of the longer arm exceeds the length of the ſhorter.

2. If an axle turns upon its gudgeons, and is fixt into a wheel; and if a rope that raiſes the weight coils round the axle, whilſt a man pulls a rope that was put round the wheel; the power gained will be as much as the diameter of the wheel, added to the diameter of the rope, exceeds the diameter of the axle added to the diameter of the rope that coils round it.

3. In the pullies, the power gained is equal to twice the number of pullies in the lower block, to which the weight

is ſuſpended. So that the power gained is always in proportion to the number of parts of the rope by which the lower block and weight are ſuſpended.

4. In the inclined plane, or half wedge, the power gained is as much as the length of the machine exceeds its thickneſs at the back, on which the ſtroke is given by the ſledge or mallet that drives the machine.

5. In the wedge, the power gained is as much as the length of both the ſides of the wedge, taken together, exceeds the thickneſs of its back, on which the blow is ſtruck by the hammer or mallet.

6. In the ſcrew, the power gained is as much as the circumference of the circle deſcribed by the working power, that turns the ſcrew, exceeds the diſtance between the threads or ſpirals of the ſcrew.

In the lever, the friction is nothing. In the wheel and axle it is as ſmall as the diameter of the gudgeons (added to the power required to bend the rope) is leſs than the diameter of the wheel; but

but it increaſes according to the weight with which the axle is charged. The like might be ſaid of the pullies, if they did not rub againſt one another, or againſt the ſides of the mortiſes in the block where they are placed. A new rope of 1 inch diameter, going over a pulley 3 inches diameter, and pulled with a force equal to 5 pounds, requires a force of 1 pound to bend it; and a rope two inches diameter requires four times as much force. In the inclined plane, wedge, and ſcrew, the friction is at leaſt equal to the power, becauſe they will ſuſtain the weight in any poſition when the power is taken off.

Wood greaſed, or metal oiled, have nearly the ſame friction; and the ſmoother they are their friction is the leſs. Yet metals may be ſo highly poliſhed, as to have their friction increaſed by the coheſion of their parts.

Wood ſlides eaſier upon the ground in wet weather than in dry; and eaſier than an equal weight of iron in dry weather: but iron ſlides eaſier than wood

wood in wet weather. Iron or ſteel running in braſs has the leaſt friction of any. Lead makes a great deal of reſiſtance. In wood acting upon wood, greaſe makes the motion at leaſt twice as eaſy. Wheel naves greaſed or tarred go four times as eaſy as when wet.

Smooth ſoft wood, moving upon ſmooth ſoft wood, has a friction equal to about a third part of the weight. In rough wood, the friction is almoſt equal to half the weight. In ſoft wood upon hard, or hard upon ſoft, the friction is equal to about a fifth part of the weight.

In poliſhed ſteel moving upon poliſhed ſteel or pewter, the friction is about a fourth part of the weight: on copper a fifth part, and on braſs a ſixth part of the weight. Metals of the ſame ſort have more friction than different ſorts.

In general, the friction increaſes in the ſame proportion with the weight. The friction is alſo greater with a greater

er velocity; but not ſo great, in proportion, as the increaſe of velocity.

To have the friction of machines as little as poſſible, they ought to be made of the feweſt and ſimpleſt parts. The diameters of the wheels and pullies ought to be large, and the gudgeons of the axles as ſmall as can be conſiſtent with their required ſtrength. The ſides of the pullies ought not to be all over flat, but to have a ſmall riſing in the middle, to keep them from rubbing againſt each other's ſides, and againſt the ſides of their mortiſes, at a diſtance from their axles. All the cords and ropes ought to be as pliable as poſſible; and for that end, rubbed with greaſe. The teeth of the wheels ſhould juſt fit and fill the openings, ſo as neither to be ſqueezed nor ſhake therein. All the parts which work into or upon one another ought to be ſmooth, the gudgeons ought juſt to fill their holes, and the working parts muſt be greaſed. The rounds or ſtaves of the trundles may be made to turn about upon iron ſpindles fixt in the

round end boards, which will take off a great deal of friction.

Let the ftrength of all the parts be in proportion to the ftrefs they are to bear; fo as they may laft equally well. He is by no means a perfect mechanic who does not only adjuft the ftrength to the ftrefs, but alfo contrive all the parts to laft fo as that one fhall not fail before another.

When any motion is to be long continued, contrive the machine fo, as that the working power may always move or act one way, if it can be done. For this is better and eafier performed than when the motion is interrupted, by the power's being forced to move firft one way and then another; becaufe every new change of motion requires a new additional force to effect it, and a body in motion cannot fuddenly receive a contrary motion without great violence, and danger of tearing the machine to pieces. But, when the nature of the thing requires that a motion fhould be fuddenly communicated to a body, or

fuddenly

ſuddenly ſtopt; let the force act againſt ſome ſpring, to prevent the machine's being damaged by a ſudden jolt.

When a machine is moved by two handles or winches on the ends of an axle; the handles are ſo placed, as that when one is up the other is down: which is the worſt way poſſible of placing them, ſave that of their being both up or down together. For, when a man raiſes a weight by means of turning a winch, he loſes half his force when the winch is upward, becauſe he puſhes himſelf as much backward as he puſhes the winch forward; and when the handle of the winch is down, directly below the axle, he loſes half his force, becauſe the winch pulls him as much toward it as he pulls it toward him: and therefore, the greateſt effect of his force on the machine is when he either pulls the winch upward, on the ſide of the axle next to him, or puſhes it downward on the ſide fartheſt from him: yet, even in theſe caſes, the pulling force is ſtronger than the puſhing.

In order to remedy this defect, as much as poſſible; the handles ſhould be ſo placed, as to ſtand at right angles to one another: and then, when there is a man at each handle, the effect of the one man's force will be greateſt when the effect of the other man's is leaſt, upon the machine. Whereas, in the common way of placing theſe handles, when the effect of one man's force is the greateſt, the other man's is ſo too; and when the effect of that man's force is the leaſt, ſo alſo is the other's; which is working at the greateſt diſadvantage poſſible.

A mechanical way of laying down the Sun's Declination right, againſt the days of the months, either on a circular or rectilineal Scale.

The Sun's declination is uſeful on many accounts; and remarkably ſo in thoſe kinds of Sun-dials which are ſo conſtructed, as that they may be ſet true,

true, in any place where the Sun ſhines, without the help of a meridian line; becauſe, when they are truly level, and have the Sun's declination laid down on their ſtiles, the dial itſelf being a circular plate placed on the middle of the ſtile, and at right angles thereto; if the dial be turned about till the ſhadow of the circular plate touches the Sun's declination for the given day, the dial will then be truly placed, and the ſtile (which will be in the plane of the meridian) will caſt a ſhadow on the true ſolar time of the day. This is the caſe in M. *Pardie's* univerſal dial, which is one of the beſt I know of; and which the reader will find particularly deſcribed in my Mechanical Lectures, ſold by Mr. *Cadell*, Bookſeller in the Strand, London.

There are Tables near the beginning of this book which ſhew the quantity of the Sun's decination at the Noon of every day of the ſecond year after Leap year, which is the neareſt mean of all the four years. But, as the declination very ſeldom comes to integral degrees

at Noon, it is difficult by theſe Tables to know at what time of the day the declination will amount to compleat degrees without fractions; and conſequently it is difficult to lay down the whole degrees thereof againſt the proper times of the days of the year, in a ſcale of months; although every day in the ſcale ſhould be divided into four equal parts, each whereof contains 6 hours.

To avoid this difficulty, I have calculated the following Table, for ſhewing the times, to the neareſt hour of the day, when the Sun's declination amounts to compleat degrees without fractions. Thus, ſuppoſing it was required to find on what days of the year, and at what hours of theſe days, the Sun's declination was juſt 9 degrees? Look for 9 in the declination columns, and againſt it you will find April 12, at 16 hours (reckoned forward from the Noon of the day) Auguſt 30th, at 0 hours (or at Noon) October 16th at 3 hours paſt Noon,

Noon, and Feb. 25th at 1 hour paſt Noon.

Now, if the 365 days of the year be laid down on a ſcale, and each day be divided into 4 equal parts thereon, in ſhorter ſtrokes than thoſe which mark the Noons of the integral days; each ſubdiviſion by the ſhorter ſtrokes will repreſent 6 hours, and any one may truſt to the accuracy of his eye in placing the diviſions for the whole degrees of declination at or between theſe ſubdiviſions of the days, as they are ſhewn by the Table to be at 0 hours (or Noon) 6 hours, 12, or 18 hours after Noon; or at any time ſooner or later.

A Table

A Table shewing at what times the Sun's declination is whole degrees, and his Place in the Ecliptic at those times.

Decl. N.	North Declination increases in			Sun's Place in the Ecliptic.			Decl. N.	North Declination decreases in			Sun's Place in the Ecliptic.		
Deg.	Mon.	D.	H.	S.	°	′	Deg.	Mon.	D.	H.	S.	°	′
0	Mar.	20	5	♈	0	0	½	June	21	5	♋	0	0
1		22	18		2	33	23	July	3	1		11	19
2		25	7		5	2	22		12	2		19	56
3		27	20		7	33	21		18	8		25	56
4		30	10		10	5	20		23	12	♌	0	52
5	Apr.	2	11		12	39	19		28	2		5	13
6		4	14		15	12	18	Aug.	1	4		9	9
7		7	7		17	49	17		5	0		12	48
8		9	23		20	27	16		8	13		16	14
9		12	16		23	7	15		11	23		19	29
10		15	11		25	50	14		15	5		22	37
11		18	7		28	37	13		18	8		25	38
12		21	6	♉	1	27	12		21	9		28	33
13		24	7		4	22	11		24	7	♍	1	23
14		27	8		7	23	10		27	4		4	10
15		30	13		10	31	9		30	0		6	53
16	May	3	23		13	46	8	Sept.	1	8		9	33
17		7	11		17	12	7		4	11		12	11
18		11	7		20	51	6		7	4		14	48
19		15	9		24	47	5		9	18		17	21
20		19	20		29	8	4		12	9		19	55
21		25	1	♊	4	4	3		15	0		22	27
22		31	7		10	4	2		17	13		24	58
23	June	9	7		18	41	1		20	3		27	27
½		21	5	♋	0	0	0		22	17	♎	0	0

The

The Table concluded.

Decl. S.	South Declination increases in			Sun's Place in the Ecliptic.			Decl. S.	South Declination decreases in			Sun's Place in the Ecliptic.		
Deg.	Mon.	D.	H.	S.	°	′	Deg.	Mon.	D.	H.	S.	°	′
0	Sept.	22	17	♎	0	0	½	Dec.	21	9	♑	0	0
1		25	9		2	33	23	Jan.	1	6		11	19
2		27	20		5	2	22		9	16		19	56
3		30	9		7	33	21		15	14		25	56
4	Oct.	2	23		10	5	20		20	10	♒	0	52
5		5	13		12	39	19		24	16		5	13
6		8	4		15	12	18		28	13		9	9
7		10	18		17	49	17	Feb.	1	4		12	48
8		13	11		20	27	16		4	13		16	14
9		16	3		23	7	15		7	18		19	29
10		18	21		25	50	14		10	19		22	37
11		21	16		28	37	13		13	20		25	38
12		24	11	♏	1	27	12		16	17		28	33
13		27	9		4	22	11		19	13	♓	1	33
14		30	10		7	23	10		22	8		4	10
15	Nov.	2	13		10	31	9		25	1		6	53
16		5	19		13	46	8		27	16		9	33
17		9	5		17	12	7	Mar.	2	7		12	11
18		12	20		20	51	6		4	22		14	48
19		16	17		24	47	5		7	12		17	21
20		21	1		29	8	4		10	2		19	55
21		25	21	♐	4	4	3		12	14		22	27
22	Dec.	1	19		10	4	2		15	4		24	58
23		10	6		18	41	1		17	15		27	27
½		21	9	♑	0	0	0		20	5	♈	0	[illegible]

A Table

A Table ſhewing the Latitudes and Longitudes of a great many remarkable Places; and what the times are at London *when it is Noon at thoſe places.*

N. *ſignifies North Latitude,* S. *South Latitude;* E. *Eaſt Longitude,* W. *Weſt Longitude, from the meridian of* London: F. *Forenoon, and* A. *Afternoon, at* London.

Noon at	Latitude. °	′		Longitude. °	′		Time at London. H.	M.	
Aberdeen	57	10	N.	1	45	W.	XII	7	A.
Abo	60	30	N.	21	30	E.	X	34	F.
Adrianople	42	00	N.	26	30	E.	X	14	F.
Aleppo	36	30	N.	37	40	E.	IX	29	F.
Algiers	36	40	N.	3	20	E.	XI	47	F.
Amſterdam	52	20	N.	4	30	E.	XI	42	F.
Annapolis Royal	45	00	N.	64	00	W.	IV	16	A.
Archangel	64	34	N.	39	00	E.	IX	24	F.
Aſtracan	47	00	N.	50	00	E.	VIII	40	F.
Azoph	47	15	N.	44	00	E.	IX	4	F.
Bagdat	33	20	N.	43	00	E.	IX	8	F.
Barcelona	41	20	N.	2	00	E.	XI	52	F.
Baſil	47	40	N.	7	40	E.	XI	29	F.
Batavia	6	00	S	106	00	E.	IV	56	F.
Bencoolen	4	00	S	101	00	E.	V	16	F.
Berlin	52	33	N.	13	31	E.	XI	6	F.
Bern	57	00	N.	7	20	E.	XI	31	F.
Bologna	44	29	N.	11	26	E.	XI	14	F.
Bombay	18	30	N.	72	00	E.	VII	12	F.
Boſton	42	24	N.	71	00	W.	IV	44	A.
Bridge Town	13	00	N.	59	00	W.	III	56	A.
Briſtol	51	30	N.	2	40	W.	XII	11	A.
Bruſſels	51	00	N.	4	6	E.	XI	44	F.
Buda	47	40	N.	19	20	E.	X	43	F.
Buenos Ayres	34	35	S.	58	26	W.	III	54	A.
Cadiz	36	31	N.	5	56	W.	XII	24	A.
Cairo (grand)	30	2	N.	31	31	E.	IX	54	F.

Candy

The Table continued.

Noon at	Latitude. ° ′	Longitude ° ′	Time at London. H. M.
Candy, in *Ceylon*	8 00 N.	79 00 E.	VI 44 F.
Cape good Hope	33 55 S.	18 35 E.	X 46 F.
Canton	23 25 N.	112 30 E.	IV 30 F.
Cape Horn	57 30 S.	80 00 W.	V 20 A.
Carthagena	11 00 N.	77 00 W.	V 8 A.
Cayenne	5 00 N.	53 00 W.	III 32 A.
Charles Town	32 30 N.	79 00 W.	V 16 A.
Conſtantinople	41 00 N.	29 00 E.	X 4 F.
Copenhagen	55 41 N.	12 50 E.	XI 9 F.
Corke	51 40 N.	8 25 W.	XII 34 A.
Cracow	50 10 N.	19 55 E.	X 40 F.
Damaſcus	33 15 N.	37 20 E.	IX 30 F.
Dantzick	18 36 N.	54 22 E	VIII 23 F.
Delly	28 00 N.	78 30 E.	VI 46 F.
Domingo, St.	18 20 N.	70 00 W.	IV 40 A.
Dreſden	51 00 N.	13 36 E.	XI 6 F.
Dublin	53 16 N.	6 25 W.	XII 26 A.
Edinburgh	55 58 N.	3 00 W.	XII 12 A.
Erzerum	39 56 N.	48 31 E.	VIII 46 F.
Exeter	50 44 N.	3 40 W.	XII 15 A.
Fez	33 30 N.	6 00 W.	XII 24 A.
Florence	43 46 N.	11 7 E.	XI 16 F.
Geneva	46 12 N.	6 25 E.	XI 34 F.
Genoa	44 30 N.	9 30 E.	XI 22 F.
Glaſgow	55 50 N.	4 8 W.	XII 16 A.
Goa	15 31 N.	73 50 E.	VII 5 F.
Hague	52 10 N.	4 00 E.	XI 48 F.
Hamburgh	54 00 N.	9 40 E.	XI 21 F.
Hanover	52 32 N.	9 35 E.	XI 22 F.
Havannah	23 00 N.	84 00 W.	V 36 A.
Helena, St.	16 00 S.	6 00 W.	XII 24 A.
Jago, St. *Jamaica*	18 20 N.	76 30 W.	V 6 A.
Jago, St. *Cuba*	20 00 N.	76 31 W.	V 6 A.
James Town	37 30 N.	76 00 W.	V 4 A.
Jeruſalem	31 50 N.	35 25 E.	IX 38 F.
Iſpahan	32 25 N.	52 55 E.	VIII 28 F.
Kingſton, *Jamaica*	17 30 N.	77 00 W.	V 8 A.
Leghorn	43 30 N.	11 00 E.	XI 16 F.
Leyden	52 12 N.	4 00 E.	XI 44 F.
Lima	12 1 S.	81 34 W.	V 24 A.
Liſbon	38 42 N.	9 25 W.	XII 38 A.

The Table continued.

Noon at	Latitude. ° ′			Longitude. ° ′			Time at London. H.	M.	
London	51	30	N.	0	00		XII	Noon	
Louiſbourg	45	54	N.	59	55	W.	III	50	A.
Madrid	40	25	N.	3	50	W.	XII	15	A.
Mahon Port	39	50	N.	4	6	E.	XI	44	F.
Malacca	2	12	N.	102	9	E.	V	12	F.
Malaga	36	40	N.	4	45	W.	XII	19	A.
Mantua	42	20	N.	11	15	E.	XI	16	F.
Mecca	21	20	N.	43	30	E.	IX	6	F.
Mexico, *Amer.*	20	00	N	103	35	W.	VI	54	A.
Milan	45	25	N.	9	25	E.	XI	22	F.
Moſcow	55	45	N.	37	51	E.	IX	29	F.
Nanking	32	00	N.	118	30	E.	IV	6	F.
Naples	40	51	N.	14	19	E.	XI	3	F.
Norwich	52	40	N.	1	26	E.	XI	54	F.
Nurimburg	49	27	N.	11	9	E.	XI	15	F.
Ormus	27	30	N.	56	00	E.	VIII	16	F.
Oſnaburg	52	31	N.	7	40	E.	XI	29	F.
Oxford	51	45	N.	1	15	E.	XI	55	F.
Palermo	38	30	N.	13	00	E.	XI	8	F.
Palmyra	33	00	N.	39	00	E.	IX	24	F.
Panama	9	00	N.	82	00	W.	V	28	A.
Paris	48	50	N.	2	25	E.	XI	50	F.
Pegu	17	30	N.	97	00	E.	V	32	F.
Peking	39	54	N.	116	28	E.	IV	14	F.
Perth	56	25	N.	3	10	W.	XII	13	A.
Peterſburg	59	58	N.	30	20	E.	IX	58	F.
Philadelphia	40	50	N.	74	00	W.	IV	56	A.
Plymouth	50	26	N.	4	27	W.	XII	18	A.
Pondicherry	11	56	N.	79	53	E.	VI	40	F.
Porto Bello	9	33	N.	79	50	W.	V	19	A.
Port Royal, *Jam.*	17	30	N.	77	00	W.	V	8	A.
Portſmouth	50	48	N.	1	6	W.	XII	6	F.
Prague	50	00	N.	14	20	E.	XI	3	F.
Preſburg	48	20	N.	17	30	E.	X	50	F.
Quebec	46	55	N.	69	48	W.	IV	39	A.
Rhodes	36	20	N.	28	00	E.	X	8	F.
Rome	41	54	N.	12	30	E.	XI	10	F.
Rotterdam	52	00	N.	4	20	E.	XI	43	F.
Saliſbury	51	6	N.	1	15	W.	XII	5	A.
Samarcand	40	00	N.	66	00	E.	VII	36	F.
Scanderoon	36	15	N.	37	00	E.	IX	32	F.

The Table concluded.

Noon at	Latitude ° ′	Longitude ° ′	Time at London. H. M.
Seville	37 15 N.	6 00 W.	XII 24 A.
Siam	14 18 N.	100 50 E.	V 17 F.
Stockholm	59 20 N.	19 25 E.	X 42 F.
Straſburg	48 35 N.	7 51 E.	XI 29 F.
Surat	21 10 N.	72 25 E.	VII 10 F.
Surinam	6 30 N.	56 00 W	III 44 A.
Syracuſe	37 25 N.	15 5 E.	XI 00 F.
Tobolſki	58 12 N.	63 10 E.	VII 27 F.
Toledo	39 50 N.	3 15 W.	XII 13 A.
Tripoli	32 54 N.	13 10 E.	XI 7 F.
Turin	45 5 N.	7 45 E	XI 29 F.
Valladolid	41 36 N.	4 50 W.	XII 19 A.
Venice	45 25 N.	12 4 E.	XI 12 F.
Vera Cruz, *Amer.*	18 30 N.	100 00 W.	VI 40 A.
Vienna	48 13 N.	16 27 E.	X 54 F.
Ulm	48 24 N.	10 00 E.	XI 20 F.
Upſal	59 52 N.	17 50 E.	X 49 F.
Uraniburg	55 54 N.	12 51 E.	XI 9 F.
Williamſburg	37 20 N.	76 30 W.	V 6 A.
Worceſter	52 15 N.	2 15 E.	XI 51 F.
York, *England*	54 00 N.	0 50 W.	XII 3 A.
New York, *Am.*	41 00 N.	72 30 W.	IV 50 A.
Zell	52 52 N.	10 00 E.	XI 20 F.
Zurick	47 52 N.	8 30 E.	XI 26 F.

Wherever XII is found in this Table, it is to be underſtood to mean Mid-day, or Noon, at London; and the minutes which follow it are the number of minutes after Mid-day, at London, when it is Noon at the place againſt which XII is found.

Beſides the uſe of this Table in ſhewing the Latitudes and Longitudes of a

great number of places, it is uſeful to thoſe who make Sun-dials: For, if a dial be made for the Meridian of London; and as many places as the artiſt can find room for, be marked on the dial, againſt the like hours which front them in the Table; the ſhadow of the Stile will fall upon theſe places when it is Noon at them reſpectively.

And, at any time when the Sun ſhines on the dial, if the time thereon be counted from the ſhadow to any given place, it will ſhew the time then at that place; which time will be before Noon, if the ſhadow be not come to the place; or Afternoon, if the ſhadow be paſt the place; juſt as much as the interval on the dial is, between the given place and the ſhadow of the Stile.

If the dial be made for any place whoſe Longitude is Eaſt from the Meridian of London, all the places to be marked upon the dial muſt be ſet forwarder than the hours which ſtand againſt them in the Table, by four minutes of time for every degree of Longitude

tude that the place has for which the dial is to ſerve. But if the Longitude of the place for which the dial is made is Weſt from the Meridian of London; all the places to be marked on the dial muſt be ſet backward from the times againſt which they ſtand in the Table, by four minutes of time of every degree of the place's Longitude. And then, when the ſhadow of the Stile falls upon theſe places, it will be Noon at them; which will be ſooner or later than at the place of the dial, according as the Longitudes of theſe places are Eaſt or Weſt from the place of the dial.

A Table

A Table for comparing the English Avoirdupoise pound with the Foreign pound weight.

London Pound	1.0000	Hamburgh	1.0865
Antwerp	1.04	Lisbon	1.135
Amsterdam	1.1111	Leghorn	0.75
Abeville	1.0989	Norimberg	1.1363
Ancona	0.78	Naples	0.71
Avignon	0.8928	Paris	1.1235
Bourdeaux	1.0989	Prague	1.2048
Bologna	0.8	Placentia	0.72
Bruges	1.0204	Rochelle	0.8928
Calabria	0.73	Rome	0.7874
Calais	0.9345	Rouen	1.1089
Dieppe	1.0989	Seville	0.9259
Dantzic	0.862	Tholouse	0.8928
Ferrara	0.75	Turin	0.82
Flanders	0.9433	Venice	1.06
Geneva	1.07	Vienna	1.23
Genoa, gross	0.7		

A Table for comparing the English Foot with Foreign measures, in English Inches.

	Inches.		Inches.
English foot	12.000	English yard	36.000
Amsterdam	11.172	English ell	45.000
Paris	12.788	Scotch ell	45.000
Rheinland	12.362	Paris aune	46.786
Scotch	12.061	Lyons aune	46.570
Dantzic	11.297	Geneva aune	44.760
Swedish	11.692	Amsterdam ell	26.800
Brussels	10.828	Danish ell	24.930
Lyons	13.458	Swedish ell	23.380
Bononian	14.938	Norway ell	24.510
Milan foot	15.631	Seville vara	33.127
Roman palm	8.779	Madrid vara	39.166
Naples palm	10.384	Portugal vara	44.031

Antwerp

Antwerp ell	27.170	Portugal cavedo	27.354
Bruſſels ell	27.260	Old Roman foot	11.632
Bruges ell	27.550	Perſian ariſh	38.364
Bononian brace	25.200	The ſhort pike of Conſtantinople	25.576
Romiſh brace	30.730		
Florence brace	22.910	The long pike	27.920

The Weight and Value of Gold and Silver Coins.

A Troy pound of gold is worth 48 pound ſterling.

A Troy ounce is worth 4 pounds ſterling. A penny weight is worth four ſhillings, and a grain is worth two pence; in coinage ſtandard.

A Troy pound of ſilver (coinage ſtandard) is worth 3 pounds ſterling: an ounce is worth 5 ſhillings; a penny weight is worth 3 pence, and a grain is worth half a farthing.

A five Moidore piece weighs 1 ounce, 14 penny weights, 15 grains. A 3 pound 12 ſh. piece weighs 18 penny-weight, 12 grains. A Guinea 5 penny-weight, 9 grains. A Moidore 6 penny-weight, 22 grains; and a Piſtole 4 p. w. 8 gr.

The

The proportion of Alloy in coinage.

The ſtandard of ſterling ſilver is 11 ounces 2 penny weight of pure ſilver, and 18 penny-weights of copper.

The ſtandard of ſterling gold is 11 Troy ounces of pure gold and 1 ounce of Alloy.

Our gold is of equal fineneſs with the Spainiſh, French, and Flemiſh; but our ſilver coin has leſs alloy in it than either French or Dutch.

Jewiſh weights reduced to Engliſh Troy weight.

A Shekel, 9 penny-weight, 2.57 grains. An hundred Shekels, or 3 lb. 9 oz. 10 p. w. 17 grains, make a Manch: and 50 Manches, or 109 lb. 8 oz. 15 p. w. 10 gr. make a Talent.

Jewiſh

Jewish-Dry measure reduced to English Corn-measure.

A Cab, $2\frac{5}{6}$ pints. An Omer, $5\frac{5}{10}$ pints. A Seah, 1 peck 1 pint. An Ephah, 3 pecks 3 pints. A Lethech, 16 pecks; and an Homer Choron 32.

Jewish Liquid-measure reduced to English.

A Log, 3 quarters of a pint. A Cab, 3 pints. A Hin, 1 gallon 2 pints. A Seah, 2 gall. 4 pints. A Bath or Ephah, 7 gallons 4 pints. A Coron, or Homer, 75 gallons 5 pints; all in wine measure.

Jewish money reduced to English.

A Gerah, 1.36 *d.* A Bekah, 1 *sh.* 1.7 *d.* A Shekel, 2 *sh.* 3.37 *d.* A Mina, 6 *l.* 16 *s.* 10.5 *d.* A Talent of silver, 342 *l.* 3 s. 9 *d.* A Shekel of gold, 1 *l.* 16 *s.* *d.* A Talent of gold, 5475 *l.*

A Table shewing the Interest of any sum of money, from a Million to a Pound, for any number of days, at any rate of Interest.

Sum.	L.	s.	d.	q.	Sum.	L.	s.	d.	q.
1000000	2739	14	6	0.99	1000	2	14	9	2.14
900000	2465	15	0	3.29	900	2	9	3	2.12
800000	2191	15	7	1.59	800	2	3	10	0.11
700000	1917	16	1	3.89	700	1	18	4	1.10
600000	1643	16	8	2.19	600	1	12	10	2.80
500000	1369	17	3	0.49	500	1	7	5	3.70
400000	1095	17	9	2.95	400	1	1	11	0.50
300000	821	18	4	1.09	300	0	16	5	1.40
200000	547	18	10	3.40	200	0	10	11	2.30
100000	273	19	5	1.70	100	0	5	5	3.15
90000	246	11	6	0.32	90	0	4	11	5.71
80000	219	3	6	0.96	80	0	4	4	2.41
70000	191	15	7	1.59	70	0	3	10	0.11
60000	164	7	8	0.22	60	0	3	3	1.81
50000	136	19	8	2.85	50	0	2	8	3.51
40000	109	11	9	1.48	40	0	2	2	1.21
30000	82	3	10	0.11	30	0	1	7	2.90
20000	54	15	10	2.74	20	0	1	1	0.60
10000	27	7	11	1.37	10	0	0	6	2.30
9000	24	13	1	3.23	9	0	0	5	3.67
8000	21	18	4	1.10	8	0	0	5	1.40
7000	19	3	6	2.96	7	0	0	4	2.41
6000	18	8	9	0.82	6	0	0	3	3.76
5000	13	13	11	2.58	5	0	0	3	1.15
4000	10	19	2	0.55	4	0	0	2	2.52
3000	8	4	4	2.41	3	0	0	1	3.80
2000	5	9	7	0.27	2	0	0	1	1.26
1000	2	14	9	2.14	1	0	0	0	2.63

Multiply

Multiply the ſum by the number of days, and the product thereof by the rate of Intereſt *per* Cent; then cut off the two laſt figures to the right hand, and enter the Table with what remains to the left; againſt which numbers collected, you have the Intereſt for the given ſum.

EXAMPLE.

Qu. *What is the Intereſt of* 100 l. *at* 5 *per Cent. for* 365 *days.*

Number of days	365	
multiply by	100 l.	
the product is	36500	
which multiplied by	5	Rate *p.* Cent.
makes	1825'00	

Then, in the Table,

			l.	s.	d.	q. parts
Againſt	1000	is	2	14	9	6.14
	800		2	3	10	0.11
	20		0	1	1	0.60
	5		0	0	3	1.15
	1825	*Anſw.*	5	0	0	0.00

Juſt 5 Pounds: and in the ſame way may the Intereſt of any other given number of Pounds may be found for any given number of Days.

The decimals are 100th parts of a farthing.

TABLES

SHEWING

The preſent Value of ANNUITIES,

AT

The moſt common Rates of INTEREST.

BY MR. DE MOIVRE.

TABLE

TABLE I.

The present Value of an Annuity of one Pound, for any number of years not exceeding 100, *Interest at* 3 per Cent.

Years.	Value.	Years.	Value.	Years.	Value.	Years.	Value.
1	0.9709	26	17.8768	51	25.9512	76	29.8076
2	1.9135	27	18.3270	52	26.1662	77	29.9103
3	2.8286	28	18.7641	53	26.3750	78	30.0100
4	3.7170	29	19.1884	54	26.5777	79	30 1068
5	4.5797	30	19.6004	55	26.7744	80	30.2008
6	5.4172	31	20.0004	56	26.9655	81	30.2920
7	6.2303	32	20.3887	57	27.1509	82	30.3806
8	7.0197	33	20.7658	58	27.3310	83	30.4666
9	7.7861	34	21.1318	59	27.5015	84	30.5501
10	8.5302	35	21.4872	60	27.6756	85	30 6311
11	9.2526	36	21.8323	61	27.8404	86	30.7099
12	9.9540	37	22.1672	62	28.0003	87	30.7863
13	10.6350	38	22.4925	63	28.1557	88	30.8605
14	11.2961	39	22.8082	64	28.3065	89	30.9325
15	11.9379	40	23.1148	65	28.4529	90	31.0024
16	12.5611	41	23.4124	66	28.7950	91	31.0703
17	13.1611	42	23.7014	67	28.7330	92	31.1362
18	13.7535	43	23.9819	68	28.8670	93	31.2001
19	14.3238	44	24.2543	69	28.9971	94	31.2622
20	14.8775	45	24.5187	70	29.1234	95	31.3224
21	15.4150	46	24.7754	71	29.2460	96	31.3809
22	15.9369	47	25.0247	72	29.3651	97	31.4377
23	16.4436	48	25.2667	73	29.4807	98	31.4928
24	16.9355	49	25.5017	74	29.5929	99	31.5463
25	17.4131	50	25.7298	75	29.7018	100	31.5984

The value of the Perpetuity is $33\frac{1}{3}$ *Years Purchase.*

TABLE

TABLE II.

The present Value of an Annuity of one Pound, for any number of years not exceeding 100, *Interest at* $3\frac{1}{2}$ per Cent.

Years.	Value.	Years.	Value.	Years.	Value.	Years.	Value.
1	0.9662	26	16.8904	51	23.6286	76	26.4799
2	1.8997	27	17.2854	52	23.7958	77	26.5506
3	2.8016	28	17.6670	53	23.9573	78	26.6190
4	3.6731	29	18.0358	54	24.1133	79	26.6850
5	4.5151	30	18.3920	55	24.2641	80	26.7488
6	5.3286	31	18.7363	56	24.4097	81	26.8104
7	6.1145	32	19.0689	57	24.5504	82	26.8700
8	6.8740	33	19.3902	58	24.6864	83	26.9275
9	7.6077	34	19.7007	59	24.8178	84	26.9831
10	8.3166	35	20.0007	60	24.9447	85	27.0368
11	9.0015	36	20.2905	61	25.0674	86	27 0887
12	9.6633	37	20.5705	62	25.1859	87	27.1388
13	10.3027	38	20.8411	63	25.3004	88	27.1873
14	10.9205	39	21.1025	64	25.4110	89	27.2341
15	11.5174	40	21.3551	65	25.5178	90	27.2793
16	12.0941	41	21.5991	66	25.6211	91	27.3230
17	12.6513	42	21.8349	67	25.7209	92	27.3652
18	13.1897	43	22.0627	68	25.8173	93	27.4060
19	13.7098	44	22.2828	69	25 9104	94	27.4454
20	14.2124	45	22.4955	70	26.0004	95	27 4835
21	14.6980	46	22.7009	71	26.0873	96	27.5203
22	15.1671	47	22.8994	72	26.1713	97	27.5558
23	15.6204	48	23.0912	73	26.2525	98	27.5902
24	16.0584	49	23.2766	74	26.3309	99	27.6234
25	16.4815	50	23.4556	75	26.4067	100	27.6554

The value of the Perpetuity is $28\frac{1}{7}$ *years purchase.*

TABLE III.

The present Value of an Annuity of one Pound, for any number of years under 100, *Interest at* 4 per Cent.

Years.	Value.	Years.	Value.	Years.	Value.	Years.	Value.
1	0.9615	26	15.9827	51	21.6714	76	23.7311
2	1.8860	27	16.3295	52	21.7475	77	23.7799
3	2.7750	28	16.6630	53	21.8726	78	23.8268
4	3.6298	29	16.9837	54	21.9929	79	23.8720
5	4.4518	30	17.2920	55	22.1086	80	23.9153
6	5.2421	31	17.5884	56	22.2198	81	23.9571
7	6.0020	32	17.8775	57	22.3267	82	23.9972
8	6.7327	33	18.1476	58	22.4295	83	24.0357
9	7.4353	34	18.4111	59	22.5284	84	24.0728
10	8.1108	35	18.6646	60	22.6234	85	24.1085
11	8.7604	36	18.9082	61	22.7148	86	24.1428
12	9.2850	37	19.1425	62	22.8027	87	24.1757
13	9 9856	38	19.3678	63	22.8872	88	24.2074
14	10 5631	39	19.5844	64	22.9685	89	24.2379
15	11.1183	40	19.7927	65	23.0466	90	24.2672
16	11.6522	41	19.9930	66	23.1218	91	24.2954
17	12.1656	42	20.1856	67	23.1940	92	24.3225
18	12.6592	43	20.3707	68	23.2635	93	24.3486
19	13.1339	44	20.5488	69	23.3302	94	24.3736
20	13.5903	45	20.7200	70	23.3945	95	24.3977
21	14.0291	46	20.8846	71	23.4562	96	24.4209
22	14.4511	47	21.0429	72	23.5156	97	24 4431
23	14.8568	48	21.1951	73	23.5727	98	24.4646
24	15.2469	49	21.3414	74	21.6276	99	24 4851
25	15.6220	50	21.4821	75	23.6804	100	24.5049

The present Value of an Annuity of one Pound, to continue so long as a Life of a given Age is in being, Interest being estimated at 3 per Cent. TABLE I.

Age.	Value.	Age.	Value.	Age.	Value.	Age.	Value.
1	15.05	26	17.50	51	12.26	76	4.05
2	16.62	27	17.33	52	12.00	77	3.63
3	17.83	28	17.16	53	11.73	78	3.21
4	18.46	29	16.98	54	11.46	79	2.78
5	18.90	30	16.80	55	11.18	80	2.34
6	19.33	31	16.62	56	10.90	81	1.89
7	19.60	32	16.44	57	10.61	82	1.43
8	19.74	33	16.25	58	10.32	83	0.96
9	19.87	34	16.06	59	10.03	84	0.49
10	19.87	35	15.86	60	9.73	85	0.00
11	19.74	36	15.67	61	9.42	86	0.00
12	19.60	37	15.46	62	9.11		
13	19.47	38	15.26	63	8.79		
14	19.33	39	15.05	64	8.46		
15	19.19	40	14.84	65	8.13		
16	19.05	41	14.63	66	7.79		
17	18 90	42	14.41	67	7.45		
18	18.76	43	14.19	68	7.10		
19	18.61	44	13.96	69	6.75		
20	18.46	45	13.73	70	6.38		
21	18.30	46	13.49	71	6 01		
22	18.15	47	13.25	72	5 63		
23	17.99	48	13.01	73	5.25		
24	17.83	49	12.76	74	4.85		
25	17.66	50	12.51	75	4.45		

TABLE II.

The present Value of an Annuity of one Pound, to continue so long as a Life of a given Age is in being, Interest being estimated at $3\frac{1}{2}$ per Cent.

Age.	Value.	Age.	Value.	Age.	Value.	Age.	Value.
1	14.16	26	16 28	51	11 69	76	3.98
2	15.53	27	16.13	52	11.45	77	3.57
3	16.56	28	15.98	53	11.20	78	3.16
4	17.09	29	15.83	54	10.95	79	2.74
5	17.46	30	15.68	55	10.69	80	2.31
6	17.82	31	15.53	56	10.44	81	1 87
7	18 05	32	15.37	57	10.18	82	1.42
8	18.16	33	15.21	58	9.91	83	0 95
9	18.27	34	15.05	59	9.64	84	0.48
10	18.27	35	14.89	60	9 36	85	0.00
11	18.16	36	14.71	61	9.08	86	0.00
12	18.05	37	14.52	62	8.79		
13	17 94	38	14.34	63	8.49		
14	17.82	39	14.16	64	8.19		
15	17.71	40	13.98	65	7.88		
16	17.59	41	13.79	66	7.56		
17	17.46	42	13.59	67	7.24		
18	17.33	43	13.40	68	6.91		
19	17.21	44	13.20	69	6.57		
20	17.09	45	12.99	70	6.22		
21	16.90	46	12.78	71	5.87		
22	16.83	47	12.57	72	5.51		
23	16.69	48	12.36	73	5.14		
24	16.56	49	12.14	74	4 77		
25	16.42	50	11.92	75	4.38		

TABLE III.

The present Value of an Annuity of one Pound, to continue so long as a Life of a given Age is in being, Interest being estimated at 4 per Cent.

Age.	Value.	Age.	Value.	Age.	Value.	Age.	Value.
1	13.36	26	15 19	51	11.13	76	3.91
2	14.54	27	15.06	52	10 92	77	3.52
3	15.43	28	14.94	53	10.70	78	3.11
4	15.89	29	14 81	54	10.47	79	2.70
5	16.21	30	14.68	55	10.24	80	2.28
6	16.50	31	14 54	56	10 71	81	1.85
7	16.64	32	14 41	57	9.57	82	1.40
8	16.79	33	14 27	58	9.22	83	0.95
9	16.88	34	14.12	59	9.07	84	0.48
10	16.88	35	13.98	60	9 71	85	0 00
11	16.79	36	13.82	61	8.45	86	0.00
12	16.64	37	13.07	62	8.28		
13	16.60	38	13.52	63	8 90		
14	16.50	39	13.36	64	7.92		
15	16.41	40	13.20	65	7.63		
16	16.31	41	13.02	66	7.33		
17	16.21	42	12.85	67	7 02		
18	16.10	43	12 68	68	6.71		
19	15.99	44	12.50	69	6.39		
20	15.89	45	12 32	70	6.06		
21	15.78	46	12.13	71	5.72		
22	15.67	47	11.94	72	5.38		
23	15.55	48	11.74	73	5.02		
24	15.43	49	11 54	74	4.66		
25	15.31	50	11.34	75	4 29		

Besides the use of the first three of these Tables, as expressed by their titles, they serve likewise to resolve the questions concerning *compound Interest*: as

1. *To find the present Value of* 1000 £. *payable* 7 *years hence, at* $3\frac{1}{2}$ per Cent. From the present value of an Annuity of 1 £. certain for 7 years, which, in *Table* II. is 6,1145, I subtract the like value for 6 years, which is 5.3286; and the remainder .7859 is the value of the 7th year's rent, or of 1 £. payable after 7 years; which multiplied by 1000 gives the answer 785 £. 18 *sh.*

2. If it be asked, *what will be the* Amount *of the sum* S *in* 7 *years at* $3\frac{1}{2}$ per Cent? Having found .7859 as above, 'tis plain the amount will be $\frac{S}{.7859}$.

3. If the question is, *In what time a sum* S *will be doubled, tripled, or increased in any given* Ratio *at* 3, $3\frac{1}{2}$, &c. per Cent. I take in the proper Table two contiguous numbers, whose difference is nearest

neareſt the reciprocal of the *Ratio* given, as $\frac{1}{2}$, $\frac{1}{3}$, &c. And the year againſt the higher number is the Anſwer.

Thus, in *Table* I. againſt the years 22, 23, ſtand the numbers 15.9639 and 16.4436; whoſe difference .5067 being a little more than .5, or $\frac{1}{2}$, ſhews that in 23 years, a ſum *S* will be a little leſs than doubled, at 3 *per Cent.* compound Intereſt. And againſt the years 36 and 37 are 21.8323, and 22.1672; the difference whereof being .3349, nearly $\frac{1}{3}$, ſhews that in 14 years more it will be almoſt tripled.

If more exactneſs is required, take the adjoining difference, whoſe error is contrary to that of the difference found; and thence compute the proportional part to be added or ſubtracted. Thus, in the laſt of theſe Examples, the difference between the years 37 and 38 is .3252, which wants .0081 of .3333 ($=\frac{1}{3}$), as the other difference .3349 exceeded it by .0016. The 38th year is therefore to be divided in the Ratio of 16 to 81; that is $\frac{16}{97}$ of a year,

year, or about 2 months to be added to the 37 years.

1. *To find at what Rate of Intereſt I ought to lay out a ſum* S, *ſo as it may increaſe* $\frac{1}{3}$ *for Inſtance, or become* $\frac{4}{3}$ S *in* 7 *years.* Here the fraction I am to look for among the differences is $\frac{3}{4}$, or the decimal .75, which is not to be found in *Tab.* I. or II. till after the limited time of 7 years. But, in *Tab.* III. the numbers againſt 6 and 7 years give the difference .7599; and the Rate is 4 *per Cent.* nearly.

So far Mr. DE MOIVRE *on this Subject.*

In queſtions concerning the Values of Lives any how combined, recourſe muſt be had to Mr. *De Moivre's* laſt Edition of his Treatiſe on Annuities.

The four following Tables, and the Remarks on them, are alſo copied from Mr. DE MOIVRE'S *Book on the Doctrine of Chances.*

The

The Probabilities of Human Life, according to different Authors.

TABLE I.

By Dr. *Halley*.

Age.	Living.	Age.	Living.	Age.	Living.	Age.	Living.
1	1000	23	580	45	397	67	172
2	855	24	574	46	387	68	162
3	798	25	567*	47	377	69	152
4	760	26	560	48	367	70	142
5	732	27	553	49	357	71	131*
6	710	28	546	50	346*	72	120
7	692	29	539	51	335	73	109
8	680	30	531*	52	324	74	98
9	670	31	523	53	313	75	88*
10	661	32	515	54	302	76	78
11	653	33	507	55	292*	77	68
12	646	34	499	56	282	78	58
13	640*	35	490*	57	272	79	49*
14	634	36	481	58	262	80	41
15	628	37	472	59	252	81	34
16	622	38	463	60	242	82	28
17	616	39	454	61	232	83	23
18	610	40	445	62	222	84	19
19	604	41	436	63	212	*	*
20	598	42	427	64	202		
21	592	43	417*	65	192		
22	586	44	407	66	182		

Table II.

By Mr. *Kerſſeboom.*

Age. 0	Living. 1400	Age.	Living.	Age.	Living.	Age.	Living.
1	1125	26	760	51	495	76	160
2	1075	27	747	52	482	77	145
3	1030	28	735	53	470	78	130
4	993	29	723	54	458	79	115
5	964	30	711	55	446	80	100
6	947	31	699	56	434	81	87
7	930	32	687	57	421	82	75
8	913	33	675	58	408	83	64
9	904	34	665	59	395	84	55
10	895	35	655	60	382	85	45
11	886	36	645	61	369	86	36
12	878	37	635	62	356	87	28
13	870	38	625	63	343	88	21
14	863	39	615	64	329	89	15
15	856	40	605	65	315	90	10
16	849	41	596	66	301	91	7
17	842	42	587	67	287	92	5
18	835	43	578	68	273	93	3
19	826	44	569	69	259	94	2
20	817	45	560	70	245	95	1
21	808	46	550	71	231	96	0.6
22	800	47	540	72	217	97	0.5
23	792	48	530	73	203	98	0.4
24	783	49	518	74	189	99	0.2
25	722	50	507	75	175	100	0.0

Table

TABLE III.

By M. *de Parcieux*.

Age.	Living.	Age.	Living.	Age.	Living.	Age.	Living.
1	****	26	766	51	571	76	192
2	****	27	758	52	560	77	173
3	1000	28	750	53	549	78	154
4	970	29	742	54	538	79	136
5	948	30	734	55	526	80	118
6	930	31	726	56	514	81	101
7	915	32	718	57	502	82	85
8	902	33	710	58	489	83	71
9	890	34	702	59	476	84	59
10	880	35	694	60	463	85	48
11	872	36	686	61	450	86	38
12	866	37	678	62	437	87	29
13	860	38	671	63	423	88	22
14	854	39	664	64	409	89	16
15	848	40	657	65	395	90	11
16	842	41	650	66	380	91	7
17	835	42	643	67	364	92	4
18	828	43	636	68	347	93	2
19	821	44	629	69	329	94	1
20	814	45	622	70	310	95	0
21	806	46	615	71	291	96	*
22	798	47	607	72	271	97	*
23	790	48	599	73	251	98	
24	782	49	590	74	231	99	
25	774	50	581	75	211	100	

Table IV.

By Meſſrs. *Smart* and *Simpſon*.

Age.	Living.	Age.	Living.	Age.	Living.	Age.	Living.	Age.	Living.
1	1280 870	17	480	33	358	49	212	65	99
2	700	18	474	34	349	50	204	66	93
3	635	19	468	35	340	51	196	67	87
4	600	20	462	36	331	52	188	68	81
5	580	21	455	37	322	53	180	69	75
6	564	22	448	38	313	54	172	70	69
7	551	23	441	39	304	55	165	71	64
8	541	24	434	40	294	56	158	72	59
9	532	25	426	41	284	57	151	73	54
10	524	26	418	42	274	58	144	74	49
11	517	27	410	43	264	59	137	75	45
12	510	28	402	44	255	60	130	76	41
13	504	29	394	45	246	61	123	77	38
14	498	30	385	46	237	62	117	78	35
15	492	31	376	47	228	63	111	79	32
16	486	32	367	48	220	64	105	80	29

Remarks on theſe four Tables of the Probabilities of Human Life.

The firſt Table is that of Dr. *Halley*, compoſed from the bills of Mortality of the city of *Breſlaw*; the beſt, perhaps, as well as the firſt of its kind: and which will always do honour to the judgement

judgement and ſagacity of its excellent author.

The next is a Table of the ingenious Mr. *Kerſſeboom*, founded chiefly upon Regiſters of the *Dutch* Annuitants, carefully examined and compared for more than a century backward. And *Monſieur* de *Parcieux* by a like uſe of the liſts of the *French Tontines*, or *long Annuities*, has furniſhed us Table III; whoſe numbers were likewiſe verified upon the *Necrologies* or mortuary Regiſters of ſeveral religious houſes of both ſexes.

To theſe is added a Table of Meſſieurs *Smart* and *Simpſon* adapted particularly to the city of *London*; whoſe inhabitants, for reaſons too well known, are ſhorter lived than the reſt of mankind.

Each of theſe Tables may have its particular uſe: the ſecond or third in valuing the *better* ſort of lives, upon which one would chooſe to hold an Annuity; the *French* may ſerve for *London*, or for lives ſuch as thoſe of its inhabitants may be ſuppoſed to be:

while Dr. *Halley's* numbers, falling between the two extremes, ſeem to approach nearer to the general courſe of nature. And in caſes of combined lives, two or more of the Tables may perhaps be uſefully employed.

Beſides theſe, the celebrated Monſieur de *Buffon* * has lately given us a new Table, from the actual obſervations of Monſieur *du Pré* de St. *Maur* of the *French Academy*. This Gentleman, in order to ſtrike a juſt *mean*, takes three populous pariſhes in the city of *Paris*, and ſo many country villages as furniſh him nearly an equal number of lives: and his care and accuracy in that performance has been ſuch as to merit the high approbation of the learned editor. It was therefore propoſed to add this Table to the reſt; after having cleared its numbers of the inequalities that neceſſarily happen in fortuitous things, as well as thoſe ariſing from the careleſs manner in which *Ages* are given to the Pariſh Clerks; by which the years that

* *Hiſtoire Naturelle,* Tom. II.

are

are multiples of 10 are generally overloaded.

But this having been done with all due care, and the whole reduced to Dr. *Halley's* denomination of 1000 infants of a year old, there resulted only a mutual confirmation of the two Tables; Mr. *du Pré's* Table making the lives ſomewhat better as far as 39 years, and thence a ſmall matter worſe than they are by Dr. *Halley's*.

We may therefore retain this laſt as no bad ſtandard for mankind in *general*; till a better police, in this and other nations, ſhall furniſh the proper *Data* for correcting it,and for expreſſing the Decrements of life more accurately, and in larger numbers.

A Table

A Table; the first part whereof shews the height to which a Barometer must be raised above the plane surface of the Earth, in order that the Mercury may stand at any given height in the Tube; and the second part shews at what height the Mercury will stand in the Tube, when the Barometer is raised to any given height above the Earth's plane surface.

Part I.			Part II.	
Height of the Mercury in inches	Height of the Barometer in feet above the Earth's plane surface.		Height of the Barometer above the Earth.	Height of the Mercury in inches.
30.000		Feet 0	Feet 0	30.00
29.000		915	1000	28.91
28.000		1862	2000	27.86
27.000		2844	3000	26.85
26.000		3863	4000	25.87
25.000		4922	5000	24.93
20.000		10947	Miles 1	24.67
15.000		18715	2	20.29
10.000		29662	3	16.68
5.000		48378	4	13.72
1.000		91831	5	11.28
0.5		110547	10	4.24
0.25		129262	20	1.60
0.1	29 miles, or	153120	25	0.95
0.001	41 miles, or	216480	30	0.23
0.000	53 miles, or	279840	40	0.08

By

By the firſt part of this Table, and a common Barometer or Weather glaſs, the perpendicular height of a hill above the plane ſurface of the Earth, may be nearly found. Thus, ſuppoſe the Mercury was obſerved to ſtand at 30 inches in the tube when at the foot of the hill, and at 27 inches when carried up to the top: againſt this ſinking of three inches, you have 2844 feet (or 948 yards) for the perpendicular height of the hill. The ſecond part is too plain to need any deſcription or example.

An account of M. Villette's *concave burning Mirror.*

This Mirror is 3 feet 11 inches in diameter, and its focal diſtance is 3 feet 2 inches. It is made of copper, tin, and biſmuth.

The effect of the Sun-beams on different bodies held in its focus were as follows:

A piece

A piece of Roman tile began to melt in 3 ſeconds, and was ready to drop in 100 ſeconds.

Chalk fled away in 33 ſeconds.

A foſſil ſhell calcined in 7 ſeconds.

Copper ore vitrified in 8 ſeconds.

Iron ore melted in 24 ſeconds.

A great tooth of a fiſh melted in 33 ſeconds.

Welch aſbeſtos was a little calcined in 28 ſeconds.

A king George's halfpenny melted in 16 ſeconds.

Tin melted in 3 ſeconds, and had a hole in it in 6.

A bone calcined in 4 ſeconds, and was vitrified in 33.

A diamond weighing 4 grains loſt $\frac{7}{8}$ parts of its weight.

The ſolar beams are condenſed 1700 times in the focus of this mirror (the condenſation in the focus being as the area of the mirror is to the area of its focus) and their heat, in the focus, is 433 times as great as the heat of common fire.

The proportional breadth of each colour in the Rain-bow, ſuppoſing the whole breadth thereof to be divided into 360 *equal parts.*

The red, 45 parts; the orange, 27; the yellow, 48; the green, 60; the blue, 60; the indigo, 40; and the violet, 80.

If the flat upper ſurface of a top be divided into 360 equal parts, all around its

its edge, and be divided by 7 lines into ſo many portions or ſectors of circles, in the above proportions, and the reſpective colours be lively painted in theſe ſpaces, but ſo as the edge of each colour may be made nearly like the colour next adjoining, that the ſeparation may not be well diſtinguiſhed by the eye; and the top be made to ſpin, all theſe colours together will appear white. And if a large round black ſpot be painted in the middle, ſo as there may be only a broad flat ring of colours around it; the experiment will ſucceed the better.

Red is the leaſt refrangible of all colours, *orange* the next leaſt, *yellow* the next, *green* the next, *blue* the next, *indigo* the next, and *violet* the moſt of all.

Mr. Edward Delaval, F. R. S. has found, by experiments on melting different metals with pure glaſs, that they colour the glaſs according to their different denſities or ſpecific gravities; the moſt denſe giving a red colour to the

glaſs, and the leaſt a blue or violet. Thus, gold melted with glaſs makes it red; lead melted with glaſs, gives it an orange colour; ſilver a yellow; copper a green; and iron a blue.

Colours produced by the mixture of colourleſs fluids.

Spirit of wine mixed with ſpirit of vitriol make a *red.*
Solution of mercury mixed with oil of tartar, *orange.*
Solution of ſublimate and lime-water, *yellow.*
Tincture of roſes and oil of tartar, *green.*
Solution of copper and ſpirit of ſal-armoniac, *purple.*
Tincture of roſes and ſpirit of wine, *blue.*
Solution of ſublimate and ſpirit of ſal armoniac, *white.*
Solution of ſugar of lead and ſolution of vitriol, *black.*

Colours produced by the mixture of coloured fluids.

Tincture of ſaffron, which is yellow, mixed with tincture of red roſes, make a *green.*

Tincture of violets, which is blue, and ſpirit of ſulphur, which is brown, make a *crimſon.*

Tincture

Tincture of red roſes, which is red, and ſpirit of hartſhorn which is browniſh, make a *blue.*

Tincture of violets, which is blue, and ſolution of Hungarian vitriol, which is blue, make a *purple.*

Tincture of violets, which is blue, and ſolution of copper, which is green, make a *violet.*

Tincture of cyanus (blue-bottle flower) which is blue, and ſpirit of ſal armoniac coloured blue, make a *green.*

Solution of Hungarian vitriol, which is blue, and lixivium, which is brown, make a *yellow.*

Solution of Hungarian vitriol, which is blue, and tincture of red roſes, make a *black.*

Tincture of cyanus, which is blue, and volution of copper, which is green, make a *red.*

Colours changed, and reſtored.

Solution of copper, which is *green*, by ſpirit of nitre is made colourleſs; and is again reſtored by oil of tartar.

Limpid infuſion of galls is made *black* by a ſolution of vitriol, and tranſparent again by oil of vitriol; and then *black* again by oil of tartar.

Tincture of *red* roſes is made black by a ſolution of vitriol, and becomes *red* again by oil of tartar.

A ſlight tincture of red roſes, by ſpirit of vitriol becomes a fine *red*; then, by ſpirit of ſal armoniac turns green; and then, by oil of vitriol becomes *red* again.

Solution of verdigreaſe, which is *green* by ſpirit of vitriol becomes colourleſs; then, by ſpirit of ſal armoniac becomes *purple*; and then, by oil of vitriol becomes colourleſs again.

The quantity of Land and of Water on the Earth's ſurface.

The ſeas and unknown parts of the Earth (by a meaſurement of the beſt maps) contain 160,522,026 ſquare miles; the inhabited parts 38,990,569: Europe, 4,456,065; Aſia, 10,768,823; Africa,

Africa, 9,564,807; and America, 14,110,874. In all, 199,512,595; which is the number of ſquare miles on the whole ſurface of the Earth.

The weight of the whole Atmoſphere.

On a ſquare inch, it is 15 pounds; on a ſquare foot, 2160; on a ſquare yard, 19,440; on a ſquare mile, 60,217,344,000; and on the whole ſurface of the Earth, and Sea together, 12,014,118,565,447,680,000 pounds.

The ſurface of the body of a middle ſiz'd man is about 14 ſquare feet; and as the weight or preſſure of the air is equal to 2160 pounds on every ſquare foot on (or near) the Earth's ſurface; and as the preſſure of the air is equal in all manner of directions, its preſſure on the whole body of a middle ſiz'd man is equal to 30,240 pounds, or 13½ tons. But, becauſe the ſpring of the internal air is of equal force with the preſſure of the external, the preſſure is not felt.

The

The diameter and circumference of the visible part of a cloudy sky.

The greatest distance of the clouds in the horizon at sea is 94 miles from the observer, all around; and consequently, the whole extent or diameter of the horizon, reaching to the clouds, is 188 miles; and the circumference thereof is 590.97 miles.

The velocity of Light.

It has been proved, by the eclipses of Jupiter's Satellites, that light takes 8 minutes of time to come from the Sun to the Earth. And as the Earth's distance from the Sun is 95,000,000 miles, in round numbers, 'tis plain that the velocity of light is 11,875,000 miles in a minute, and consequently 197,916 miles in a second; which is 1,486,458 times as swift as the motion of a cannon ball, and 10,440 times as swift as the Earth moves in its annual orbit.

The

The velocity of Sound.

According to Dr. *Halley*, Mr. *Flamſteed*, and Mr. *Derham*, ſound moves 1142 feet in a ſecond of time, 68520 feet in a minute, and 778.636 miles in an hour.

Hence we may know how far a thunder cloud is from us, if we have a watch that ſhews ſeconds. Thus, ſuppoſe there were four ſeconds from the moment we ſee the flaſh of *lightning* to the moment we hear the clap of *thunder*, 'tis plain that the cloud which produced the thunder is four times 1142 feet, or 4568 feet from us; which is about four fifths of a mile.

The cauſe of the ebbing and flowing of the ſea at the ſame time on oppoſite ſides of the globe.

The reaſon why the tides riſe on the ſide of the Earth which is at any time turned towards the Moon, is plain to every one; becauſe her attraction muſt

occaſion

occaſion a ſwelling of the waters toward her on that ſide: but the cauſe of as great a ſwell, at the ſame time, on the oppoſite ſide of the Earth, which is then turned away from the Moon, has been very hard to account for; becauſe the riſing of the tide there is in a direction quite contrary to the attraction of the Moon. But this difficulty is immediately removed, when we conſider, that all bodies moving in circles have a centrifugal force, or conſtant tendency to fly off from the centers of the circles they deſcribe; and this centrifugal force is always in proportion to the diſtance of the body from the center of its orbit, and the velocity with which it moves therein.

When the body is large, the ſide of it which is fartheſt from the center of its orbit will have a greater degree of centrifugal force than the center of the body has; and the ſide of it which is neareſt the center of its orbit will have a leſs degree of centrifugal force than its center has.

As

As the Moon goes round the Earth every month in her orbit, the Earth alſo goes round an orbit every month, which is as much leſs than the Moon's orbit, as the quantity of matter in the Moon is leſs than the quantity of matter in the Earth, which is 40 times. For, by the laws of nature, when a ſmall body moves round a great one, in free and open ſpace, both theſe bodies muſt move round the common center of gravity between them.

The Moon's mean diſtance from the Earth's center is 240,000 Engliſh miles: divide therefore this diſtance by 40, the difference between the quantity of matter in the Earth and Moon; and the quotient will be 6000 miles, which is the diſtance of the common center of gravity (between the Earth and Moon) from the center of the Earth.

Now, as the Earth and Moon move round the common center of gravity between them, once every month; 'tis plain, that whilſt the Moon moves round her orbit, at 240,000 miles from the

Earth's center, the center of the Earth deſcribes a circle of 6000 miles radius, round the center of gravity between the Earth and the Moon; the Moon's attraction balancing the centrifugal force of the Earth at its center.

The diameter of the Earth is 8000 miles, in round numbers, and conſequently its ſemidiameter is 4000: ſo that the ſide of the Earth, which is at any time turned toward the Moon, is 4000 miles nearer the common center of gravity between the Earth and Moon than the Earth's center is; and the ſide of the Earth, which is then fartheſt from the Moon, is 4000 miles farther from the center of gravity between the Earth and Moon than the Earth's center is at that time.

Therefore, the radius of the circle deſcribed by the parts of the Earth which come about toward the Moon, by the Earth's diurnal motion, is 2000 miles; the radius of the circle deſcribed by the Earth's center is 6000; and the radius of the circle deſcribed by thoſe

parts

parts of the Earth which, in revolving on its axis, are furtheſt from the Moon, is 10000 miles.

The centrifugal forces of the different parts of the Earth being directly as their diſtances from the abovementioned common center of gravity, round which both the Earth and Moon move, theſe forces may be expreſſed by 2000 for the ſide of the Earth neareſt the Moon, by 6000 for the Earth's center, and by 10000 for the ſide of the Earth which is fartheſt from the Moon.

But the Moon's attraction is greateſt on the ſide of the Earth next her, where the centrifugal force or tendency to fly off from the common center of gravity (and conſequently, from the Moon) is leaſt; and therefore, the tides muſt riſe on the ſide of the Earth which is neareſt the Moon, by the exceſs of the Moon's attraction.

As her attraction balances the centrifugal force at the Earth's center, 'tis plain that the centrifugal force of the ſide of the Earth which is fartheſt f.om

the Moon is greater than her attraction; and therefore, the tides will rise as high upon that side from the Moon, by the excess of the centrifugal force, as they rise on the side next her by the excess of her attraction. And as the Earth is in constant motion on its axis, so as that any given meridian revolves from the Moon to the Moon again in 24 hours, 50½ minutes, each place will come to the two eminences of water, under and opposite to the Moon, in 24 hours, 50½ minutes, or have two tides of flood and two of ebb in that time. For, as much as the waters rise above the common level of the surface of the sea, under and opposite to the Moon, so much they must fall below that level half way between the highest places; or at 90 degrees from them.

On these principles, it is equally easy to account for the rising of the tides, at the same time, on both sides of the Earth: and this rising is made evident to sight in my Lecture on the central forces; and the principles on which it depends

depends are made obvious to the underſtandings of all the obſervers.

Surpriſing properties of numbers, placed in ſquares and circles.

I have ſeen ſeveral different kinds of (what is generally called) magic ſquares; but have lately got a magic ſquare of ſquares and a magic circle of circles of a very extraordinary kind, from Dr. BENJAMIN FRANKLIN of *Philadelphia*, with his leave to publiſh them. The magic ſquare goes far beyond any thing of the kind I ever ſaw before; and the magic circle (which is the firſt of the kind I ever heard of, or perhaps any one beſides) is ſtill more ſurpriſing. What the Doctor's rules are, for diſpoſing of the different numbers ſo, as that they ſhall have the following properties, I know nothing of: and perhaps the reaſon may be, that I have not ventured to aſk him; although I never ſaw a more communicative man in my life. The plates of theſe are at the end of the book.

PLATE I.

A magic ſquare of ſquares.

The great ſquare is divided into 256 ſmall ſquares, in which all the numbers from 1 to 256 are placed, in 16 columns, which may be taken either horizontally or vertically. The properties are as follows.

1. The ſum of the ſixteen numbers in each column, vertical and horizontal, is 2056.

2. Every half column, vertical and horizontal, makes 1028, or half of 2056.

3. Half a diagonal aſcending, added to half a diagonal deſcending, makes 2056; taking theſe half diagonals from the ends of any ſide of the ſquare to the middle thereof; and ſo reckoning them either upward, or downward; or ſidewiſe from left to right hand, or from right to left.

4. The ſame with all the parallels to the half diagonals, as many as can be drawn

drawn in the great ſquare: for any two of them being directed upward and downward, from where they begin to where they end, their ſums will make 2056. The ſame downward and upward from where they begin to where they end; or all the ſame if taken ſidewiſe to the middle, and back to the ſame ſide again.

N. B. One ſet of theſe half diagonals and their parallels, is drawn in the ſquare upward and downward. Another ſuch ſet may be drawn from any of the other three ſides.

5. The four corner numbers in the great ſquare added to the four central numbers therein, make 1028; equal to the half ſum of any vertical or horizontal column, which contains 16 numbers; and equal to half a diagonal or its parallel.

6. If a ſquare hole (equal in breadth to four of the little ſquares) be cut in a paper, through which any of the ſixteen little ſquares in the great ſquare may be ſeen, and the paper be laid on the

great

great ſquare; the ſum of all the 16 numbers, ſeen through the hole, is equal to the ſum of the ſixteen numbers in any horizontal or vertical column, *viz.* to 2056.

PLATE II.

A magic circle of circles.

This circle is compoſed of a ſeries of numbers, from 12 to 75 incluſive, divided into eight concentric circular ſpaces, and ranged in eight radii of numbers, with the number 12 in the center; which number, like the center, is common to all theſe circular ſpaces, and to all the radii.

The numbers are ſo placed, that the ſum of all thoſe in either of the concentric circular ſpaces above-mentioned, together with the central number 12, make 360; equal to the number of degrees in a circle.

The numbers in each radius alſo, together with the central number 12, make juſt 360. The

The numbers in half of any of the above circular ſpaces, taken either above or below the double horizontal line, with half the central number 12, make 180; equal to the number of degrees in a ſemi-circle.

If any four adjoining numbers be taken, as if in a ſquare, in the radial diviſions of theſe circular ſpaces; the ſum of theſe, with half the central number, make 180.

There are, moreover, included four ſets of other circular ſpaces, bounded by circles which are excentric with reſpect to the common center; each of theſe ſets containing five ſpaces. The centers of the circles which bound them are at *A*, *B*, *C*, and *D*. The ſet whoſe center is at *A* is bounded by dotted lines; the ſet whoſe center is at *C* is bounded by lines of ſhort unconnected ſtrokes; and the ſet round *D* is bounded by lines of unconnected longer ſtrokes, to diſtinguiſh them from one another. In drawing this figure by hand, the ſet of concentric circles ſhould be drawn

with black ink ; and the four different ſets of excentric circles with four kinds of ink of different colours; as blue, red, yellow, and green, for diſtinguiſhing them readily from one another.

Theſe ſets of excentric circular ſpaces interſect thoſe of the concentric, and each other: and yet, the numbers contained in each of the excentric ſpaces, taken all around through any of the 20, which are excentric, make the ſame ſum as thoſe in the concentric; namely 360, when the central number 12 is added. Their halves alſo, taken above or below the double horizontal line, with half the central number, make 180.

Obſerve, that there is not one of the numbers but what belongs at leaſt to two of the circular ſpaces; ſome to three, ſome to four, ſome to five: and yet they are all ſo placed, as never to break the required number 360, in any of the 28 circular ſpaces within the primitive circle.

To bring theſe matters in view, I have taken out all the numbers as above-

above-mentioned: and have placed them in ſeparate columns, as they ſtand around both the concentric and excentric circular ſpaces, always beginning with the outermoſt, and ending with the innermoſt of each ſet; and alſo the numbers as they ſtand in the eight radii, from the circumference to the center: the common central number 12 being placed the loweſt in each column.

1. In the eight concentric circular ſpaces.

14	72	23	65	21	67	12	74
25	63	16	70	18	68	27	61
30	56	39	49	37	51	28	58
41	47	32	54	34	52	43	45
46	40	55	33	53	35	44	42
57	31	48	38	50	36	59	29
62	24	71	17	69	19	60	26
73	15	64	22	66	20	75	13
12	12	12	12	12	12	12	12
360	360	360	360	360	360	360	360

2. In the eight radii.

14	25	30	41	46	57	62	73
72	63	56	47	40	31	24	15
23	16	39	32	55	48	71	64
65	70	49	54	33	38	17	22
31	18	37	34	53	50	69	66
67	68	51	52	35	36	19	20
12	27	28	43	44	59	60	75
74	61	58	45	42	29	26	13
12	12	12	12	12	12	12	12
360	360	360	360	360	360	360	360

3. In the five excentric circular ſpaces whoſe center is at A.	14	32	23	65	21
	63	16	70	18	68
	39	49	37	51	28
	54	34	52	43	45
	33	53	35	44	42
	48	38	50	36	59
	24	71	17	69	19
	73	15	64	22	66
	12	12	12	12	12
	360	360	360	360	360
4. In the five excentric circular ſpaces whoſe center is at B.	30	56	39	49	37
	47	32	54	34	52
	55	33	53	35	44
	38	50	36	59	29
	17	69	19	60	26
	64	22	66	20	75
	72	23	65	21	67
	25	63	16	70	18
	12	12	12	12	12
	360	360	360	360	360
5. In the five excentric circular ſpaces whoſe center is at C.	46	40	55	33	53
	31	48	38	50	36
	71	17	69	19	60
	22	66	20	75	13
	65	21	67	12	74
	16	70	18	68	27
	56	39	49	37	51
	41	47	32	54	34
	12	12	12	12	12
	360	360	360	360	360
6. In the five excentric circular ſpaces whoſe center is at D.	62	24	71	17	59
	15	64	22	66	20
	23	65	21	67	12
	70	18	68	27	61
	49	37	51	28	58
	32	54	34	52	43
	40	55	33	53	35
	57	31	48	38	50
	12	12	12	12	12
	360	360	360	360	360

If

If now, we take any four numbers, as in a ſquare form, either from N°. 1. N°. 2. (as ſuppoſe from N°. 1.) as in the margin; and add half the central number 12 to them, the ſum will be 180; equal to half the numbers in any circular ſpace, taken above or below the double horizontal line: and equal to the number of degrees in a ſemicircle. Thus, 14, 72, 25, 63, and 6, make 180.

14	72
25	63
6	

A List of the Apparatus on which Mr. Ferguson *reads his Course of twelve Lectures on Mechanics, Hydrostatics, Hydraulics, Pneumatics, Dialing and Astronomy.*

The numbers relate to the Lectures read on the machinery to which they are prefixed.

I.

Simple machines for demonstrating the powers of the lever, the wheel and axle, the pullies, the inclined plane, the wedge, and the screw.

A compound engine in which all these simple machines work together.

A working model of the great crane at *Bristol*, which is reckoned to be the best crane in Europe.

A working model of a crane that has four different powers, to be adapted to the different weights intended to be raised: invented by Mr. *Ferguson*.

A py-

A pyrometer that makes the expanſion of metals by heat viſible to the 90 thouſandth part of an inch; ſo as to be ſeen by the bare eye at two feet diſtance from the machine.

II.

Simple machines for ſhewing the center of gravity of bodies, and how far a tower may incline without danger of falling.

A double cone that ſeemingly rolls up-hill of itſelf, whilſt it is actually deſcending.

A machine made in the figure of a human creature, that tumbles backward, by continually overſetting the center of gravity.

Models of wheel carriages; ſome with broad wheels, others with narrow; ſome with large wheels, and others with ſmall: for proving experimentally which ſort is the beſt.

A machine for ſhewing what degree of power is ſufficient to draw a loaded

cart

cart or waggon up-hill; when the quantity of weight to be drawn up, and the angle of the hill's height, are known.

A machine for diminiſhing friction; and ſhewing that the friction is not in proportion to the quantity of the ſurface that either rubs or rolls; but in proportion to the weight with which the machine is loaded.

A model of a moſt curious Silk-reel, invented by Mr. *Verrier* near *Wrington* in *Somerſetſhire*.

A large working model of a Water-mill for ſawing timber.

A model of a Hand-mill for grinding corn.

A model of a Water-mill, for winnowing and grinding corn, drawing up the ſacks, and boulting the flour.

A model of Dr. *Barker's* Water-mill (for grinding corn) in which Mill there is neither wheel nor trundle.

A machine for demonſtrating that the power of the wind, on windmill ſails,

ſails, is as the ſquare of the velocity of the wind.

A model of the engine by which the piles were driven, for a foundation to the piers of *Weſtminſter* bridge.

III.

A machine for ſhewing that fluids weigh as much in their own elements as they do in air.

A machine for ſhewing that, on equal bottoms, the preſſure of fluids is in proportion to their perpendicular heights; let their quantities be ever ſo great or ever ſo ſmall.

Machines for ſhewing that fluids preſs equally in all manner of directions.

A machine for ſhewing how an ounce of water in a tube may be made to raiſe and ſupport ſixteen pound weight of lead.

A machine for ſhewing, that, at equal heights, the ſmalleſt quantity of water whatever will balance the greateſt quan-

 tity

tity whatever, if the columns join at bottom.

A machine for ſhewing how ſolid lead may be made to ſwim in water, and the lighteſt wood to ſink in water.

Machines for ſhewing and demonſtrating the hydroſtatical paradox.

A machine for demonſtrating that the weight of the quantity of water diſplaced by a ſhip is equal to the whole weight of the ſhip and cargo.

Machines for ſhewing the working of ſyphoms, and the *Tantalus*'s cup.

A large machine for ſhewing the cauſe and explaining the phenomena of ebbing and flowing wells, and of intermitting and reciprocating ſprings.

IV.

Machines for ſhewing that when ſolid bodies are immerſed and ſuſpended in fluids, the ſolid loſes as much of its weight as its bulk of the fluid weighs; and that the weight loſt by the ſolid is imparted to the fluid.

A hy-

A hydroſtatic balance, for ſhewing the ſpecific gravities of bodies, and detecting counterfeit gold or ſilver.

A working model of *Archimedes*'s ſpiral pump.

Glaſs models for ſhewing the ſtructure and operations of ſucking, forcing and lifting pumps.

A working model of an engine for extinguiſhing fire.

A working model of a quadruple Pump-mill for raiſing water by means of water turning a wheel.

A working model of the *Perſian* wheel for raiſing water.

A model of the great hydraulic engine under *London* bridge, that goes by the tides, and raiſes water by forcing pumps.

V. and VI.

An Air-pump, with a great apparatus to it, for experiments ſhewing the weight and ſpring of the air.

A Wind-gun.

VII.

A large armillary ſphere, for ſhewing the apparent motions of the Sun and Moon, with the times of their riſing and ſetting, in all latitudes, and on all the days of the year.

A wooden model of an aſtronomical clock, ſhewing the apparent motions and times of riſing and ſetting of the Sun, Moon, and Stars, with the age and phaſes of the Moon, at all times.

Another model of a clock, for ſhewing the apparent motions of the Sun and ſtars, with the times of their riſing and ſetting, and the equation of natural days.

A ſimple machine, by which all the principles of dialing are made evident to ſight.

Pardie's univerſal dial, for finding a meridian line, and ſhewing the true ſolar time of the day.

Three dials of different kinds joined together; on all of which, the time of the

the day is ſhewn by the ſhadow of one ſtile.

A collection of nine dials, all in one portable inſtrument, ſhewing the time of the day in all latitudes; and all the places of the Earth where it is day, and where it is night, at any time when the Sun ſhines on the dial; and all the places of the Earth to which the Sun is riſing, and to which it is ſetting at that time.

An univerſal dial in the form of a plain croſs.

An inſtrument for finding the true diſtances of all the Forenoon and Afternoon hours from XII, on horizontal and vertical dials, for all latitudes. And alſo for finding the hour of the day, and the variation of the compaſs, at any place; together with the Sun's declination, azimuth, amplitude, and time of riſing and ſetting, in any given latitude.

VIII.

VIII.

A whirling table, for explaining and demonſtrating the laws by which the planets move, and are retained in their orbits: that the Sun and all the planets move round their common center of gravity: that the Earth and Moon move round their common center of gravity once every month: that the Earth moves round the Sun, in common with the reſt of the planets, and turns round its own axis: that the power of gravity diminiſhes in proportion as the ſquare of the diſtance from the attracting body increaſes: that a double velocity in any orbit would require a quadruple power of gravity to retain the body in that orbit: that the ſquares of the periodical times in which the planets move round the Sun are in proportion to the cubes of their diſtances from the Sun. A plain experimental demonſtration of the doctrine of the tides; and the cauſe of their riſing equally high, at the ſame time, on oppoſite ſides of the Earth.

IX,

IX, X, XI, and XII.

A machine for ſhewing the motions of the comets.

An ORRERY, ſhewing the real motions of the planets round the Sun; the apparent ſtations, direct and retrograde motions of Mercury and Venus, as ſeen from the Earth: the different lengths of days and nights, and all the viciſſitudes of ſeaſons, ariſing from the diurnal and annual motions of the Earth: the motions and various phaſes of the Moon: the Harveſt-moon: the tides: the cauſes, times and returns of all the eclipſes of the Sun and Moon: the eclipſes of Jupiter's ſatellites, and the phenomena of Saturn's ring.

In London, any number of perſons, not leſs than twenty, who will ſubſcribe one Guinea each, may have a courſe of twelve Lectures read on the above-mentioned Apparatus, provided they agree to have at leaſt three Lectures a week;

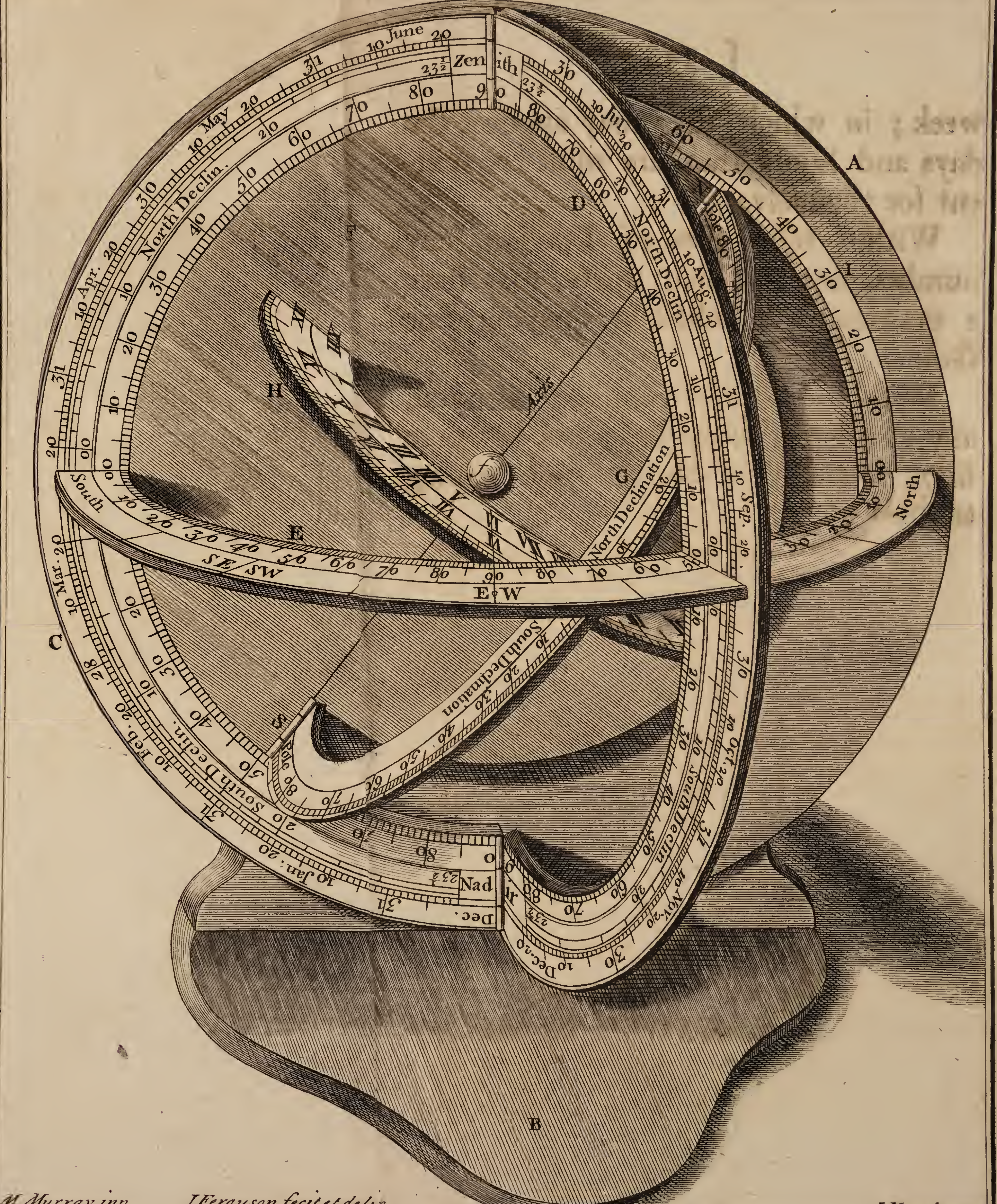
week; in which they may appoint the days and hours that are moſt convenient for themſelves.

Within ten miles of London, any number, not leſs than thirty, may have a courſe; each ſubſcriber paying one Guinea. And,

Within an hundred miles of London, any number of ſubſcribers, not leſs than ſixty, may have a courſe; each paying as above.

FINIS.

Plate I.



M. Murray inv. *J. Ferguson fecit et delin.* *J. Mynde sculp.*

Pl. II. *A Magic Square of Squares.*

200	217	232	249	8	25	40	57	72	89	104	121	136	153	168	185
58	39	26	7	250	231	218	199	186	167	154	135	122	103	90	71
198	219	230	251	6	27	38	59	70	91	102	123	134	155	166	187
60	37	28	5	252	229	220	197	188	165	156	133	124	101	92	69
201	216	233	248	9	24	41	56	73	88	105	120	137	152	169	184
55	42	23	10	247	234	215	202	183	170	151	138	119	106	87	74
203	214	235	246	11	22	43	54	75	86	107	118	139	150	171	182
53	44	21	12	245	236	213	204	181	172	149	140	117	108	85	76
205	212	237	244	13	20	45	52	77	84	109	116	141	148	173	180
51	46	19	14	243	238	211	206	179	174	147	142	115	110	83	78
207	210	239	242	15	18	47	50	79	82	111	114	143	146	175	178
49	48	17	16	241	240	209	208	177	176	145	144	113	112	81	80
196	221	228	253	4	29	36	61	68	93	100	125	132	157	164	189
62	35	30	3	254	227	222	195	190	163	158	131	126	99	94	67
194	223	226	255	2	31	34	63	66	95	98	127	130	159	162	191
64	33	32	1	256	225	224	193	192	161	160	129	128	97	96	65

B. Franklin inv. J. Ferguson delin. *J. Mynde sc.*

Pl. III. *A Magic Circle of Circles.*

B. Franklin inv. I. Ferguson delin. *J. Mynde sc.*